JHEP HYPER VERSION

Received: *October 31, 2000,* Accepted: *February 12, 2001*
Revised: *January 17, 2001*

# String loop corrections to stable non-BPS branes

**Neil D. Lambert**

*Department of Mathematics, King's College*
*The Strand, London, WC2R 2LS, England*
*E-mail:* lambert@mth.kcl.ac.uk

**Ivo Sachs**

*Theoretische Physik, Ludwig-Maximilians Universität*
*Theresienstrasse 37, 80333 Munich, Germany*
*E-mail:* ivo@theorie.physik.uni-muenchen.de

Abstract: We calculate the string loop corrections to the tachyon potential for stable non-BPS D$p$-branes on the orbifold $T^4/\mathbb{Z}_2$. We find a non-trivial phase structure and we show that, after tachyon condensation, the non-BPS D$p$-branes are attracted to each other for $p = 0, 1, 2$. We then identify the corresponding closed string boundary states together with the massless long range fields they excite. For $p = 3, 4$ the string loop correction diverge. We identify the massless closed string fields responsible for these divergencies and regularise the partition function using a Fischler-Susskind mechanism.

Keywords: D-branes, Brane Dynamics in Gauge Theories.

JHEP02(2001)018

## Contents





# 1. Introduction

Non-BPS states provide an important testing ground for various dynamical aspects of non-perturbative string theory which are not protected by supersymmetry. A peculiar feature of non-BPS branes is that they typically include tachyons among their worldvolume fields. Much has been learned about the fate of these tachyons, both with conformal field theory methods, where they correspond to marginal deformations of the open string sigma model [1]–[11] and also within open string field theory where the tachyons do not correspond to marginal deformations [12]–[22]. So far these results have mainly been restricted to tree level approximation.[1] On the other hand it is important to understand string loop corrections to these classical results, especially in view of a supergravity description of non-BPS branes [23, 24, 25, 26]. Of course, this aspect is also relevant to possible generalisations of the AdS/CFT correspondence [27] to non-supersymmetric models. Indeed, the effective field theories describing the low energy excitations of non-BPS branes are non-supersymmetric at all scales [28, 29, 30], not just in the infrared.

[1] however see [7, 8] for some aspects of tachyon condensation in the boundary state formalism.

In this paper we address the issue of open string-loop corrections to the tachyon potentials for non-BPS states. Of course, in order to have convergent loop integrals we should expand around a background which is a minimum of the tree level potential. Hence we take as our starting point stable non-BPS branes in type-II theory (see [2] and references therein). The results we obtain in this way nevertheless enable us to draw some conclusions about non-critical non-BPS branes. For example, it has been shown in [31] that, in the absence of tachyon condensates and away from the critical radius, stable non-BPS branes repel. Therefore it is not clear that a supergravity solution can be found. Here we will see that, at the critical radius, stable non-BPS D$p$-branes with $p = 0, 1, 2$ in fact choose a vacuum where their worldvolume scalars arising from the tachyon momentum modes condense. Furthermore in this vacuum the force between two stable and parallel non-BPS branes is attractive even away from the critical radius. Thus one may expect to find supergravity solutions representing large numbers of these non-BPS branes at weak string coupling.

The rest of this paper is organised as follows. In section 2 we prepare the ground by reviewing the definitions and properties of non-BPS branes relevant to our calculations. In section 3 we calculate, using a field redefinition introduced in section 2, the expression for the annulus partition function of open string theory. In section 4 we evaluate the effective potential numerically, after regularising the partition function. We also discuss the force between two parallel non-BPS branes at the minima of the effective potential for various orbifold radii. In section 5 we construct the closed string boundary states of the non-BPS branes obtained at the minimum of the effective potential. This provides a check on our work and furthermore allows us to identify the massless closed string counterterms needed to regularise the effective potential. Also, we extend the discussion of the existence of bound states for non-critical orbifolds. Finally, in section 6 we summarise and discuss the various results and some open problems.

## 2. Review of stable non-BPS branes wrapped on $T^4/\mathcal{I}_4(-1)^{F_L}$.

Let us first review some features of non-BPS branes that we will need. A non-BPS D$p$-brane of type-IIA/B string theory can be constructed by taking a D$p$-$\bar{\text{D}}p$-brane pair in type-IIB/A string theory (see [2, 32] for a review). There are then four Chan-Paton (CP) factors corresponding to the locations of the two ends points of the open strings. For the strings with both ends on the same brane the GSO projection removes the NS ground state so that the lightest modes form a maximally supersymmetric Yang-Mills U(1) $\times$ U(1) multiplet $(A_\mu, \phi^I, \lambda)$ in $p + 1$ dimensions. However, for strings that stretch between different branes, the NS ground state survives the GSO projection. Therefore the lightest modes consist of a complex tachyon $T$ and a thirty-two component massless fermion $\psi$. The next step is to mod-out the theory by $(-1)^{F_L}$, where $F_L$ is the left-moving spacetime fermion number. This will take us from type-IIB/A theory to type-IIA/B theory in the bulk. For the open strings this identifies the D$p$-brane with the $\bar{\mathrm{D}}p$-brane and projects out all but the $I$ and $\sigma_1$ CP factors. Thus the gauge group in $I$-sector is projected down to U(1) representing a single object. In the $\sigma_1$-sector the tachyon and fermions become real. This results in a non-BPS D$p$-brane ($\tilde{\mathrm{D}}p$-brane) with $p$ odd/even in type-IIA/B string theory. However, the presence of the tachyon implies that a $\tilde{\mathrm{D}}p$-brane is unstable and will decay into the vacuum.

We may construct stable $\tilde{\mathrm{D}}p$-branes by wrapping a $\tilde{\mathrm{D}}(p+4)$-brane over the orbifold $T^4/g$ where

$$g = \mathcal{I}_4(-1)^{F_L}\,, \tag{2.1}$$

and $\mathcal{I}_4 : x^i \to -x^i$, $i = 6, 7, 8, 9$. It was shown by Sen [1] that the NS-$\sigma_1$ ground state is odd under $g = \mathcal{I}_4(-1)^{F_L}$. Therefore the tachyon zero-mode is projected out whereas the modes with odd winding around the orbifold are not. In this way we see that the lightest states in the NS-$\sigma_1$ sector are the four scalar modes $\chi^i$ coming from the Fourier expansion of $T$

$$\frac{\chi^i(x^\mu)}{2i}\left(e^{iX^i/R_i} - e^{-iX^i/R_i}\right), \tag{2.2}$$

where $x^\mu$ are the non-compact coordinates along the brane. These have a mass-squared $m_i^2 = -1/2 + 1/R_i^2$ which is positive if $R_i \le \sqrt{2}$. Thus we obtain a stable $\tilde{\mathrm{D}}p$-brane in the non-compact space. Here, $p$ labels the number of non-compact extended spatial dimensions of the brane.[2] Note that, although we continue to use the term "tachyon" to describe various scalars $\chi^i$, after the orbifold projection at the critical radius these modes are in fact massless. Thus the lightest states in the $\sigma_1$-sector to survive the orbifold consist of four massless scalars $\chi^i$ and a sixteen component massless fermion $\psi_-$. In the $I$-sector the effect of the orbifold is simply to remove the gauge potentials $A_i$, $i = 6, \ldots, 9$ along the orbifold direction, and also to remove half of the fermions $\lambda_+$. In total we now find the same field content as a maximally supersymmetric Yang-Mills multiplet in $p+1$ dimensions: a vector $A_\mu$, $\mu = 0, 1, 2, \ldots, p$, $9-p$ scalars $\phi^I, \chi^i$, $I = p+1, \ldots, 5$, $i = 6, 7, 8, 9$, and two sixteen component fermions $\lambda_+, \psi_-$. Here $\pm$ labels the six-dimensional chirality of the fermions.

The low energy effective field theory for these non-BPS branes has been derived in [29, 30]. At the critical radius, the lowest order term in an $\alpha'$ expansion is

$$\begin{aligned}\mathcal{L} = \mathrm{Tr}\Big[&\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(D_\mu\phi_I)(D^\mu\phi^I) + \frac{1}{2}(D_\mu\chi_i)(D^\mu\chi^i) - ig_{\mathrm{YM}}\bar{\lambda}_+\Gamma^\mu D_\mu\lambda_+ - \\ &- ig_{\mathrm{YM}}\bar{\psi}_-\Gamma^\mu D_\mu\psi_- + g_{\mathrm{YM}}\bar{\lambda}_+\Gamma^I[\phi^I, \lambda_+] + g_{\mathrm{YM}}\bar{\psi}_-\Gamma^I[\phi^I, \psi_-] - V\Big]\end{aligned} \tag{2.3}$$

[2]Often a stable non-BPS brane with $p$ non-compact spacial dimensions is defined by taking the T-dual along the wrapped orbifold directions [31]. Then the massless scalars $\chi_i$ arise from winding modes in the orbifold directions.

where

$$V = \frac{g_{\rm YM}^2}{4}\sum_{I,J}([\phi^I,\phi^J])^2 + \frac{g_{\rm YM}^2}{2}\sum_{I,j}([\phi^I,\chi^j])^2 - \frac{g_{\rm YM}^2}{4}\sum_{i\neq j}(\{\chi^i,\chi^j\})^2\,, \tag{2.4}$$

and $D_\mu = \partial_\mu + ig_{\rm YM}[A_\mu,\ ]$. The moduli space of vacua is parameterised by constant scalar vevs which satisfy

$$[\phi^I,\phi^J] = [\phi^I,\chi^j] = \{\chi^i,\chi^j\} = 0\,, \tag{2.5}$$

for $i\neq j$. A peculiar feature of this model is that the potential (2.4) has flat directions where two or more anti-commuting tachyon vevs are turned on. Furthermore it was shown in [30] that these flat directions, which correspond to marginal deformations, persist to all orders in $\alpha'$ at tree level in open string theory. The purpose of this paper is to determine the fate of these flat directions after the inclusion of one loop open string corrections.

If we move away from the critical radius then a mass term for the $\chi^i$'s appear in the lagrangian (2.3) [29] and the flat directions are removed. In much of this paper we will be most interested in the partition function at the critical radius. At this point it is possible to perform a field redefinition that maps the tachyon vertex operator into a Wilson line [5, 2, 9]. Here we will exploit this redefinition so let us also review it.

The objects of interest are the vertex operators for the momentum modes of $T$ in the (0) and $(-1)$-pictures. We use the conventions of [29] and set $\alpha' = g_o = 1$. At the critical radius

$$V_T^{(-1)i} = -\frac{i}{\sqrt{2}}e^{-\Phi}\left(e^{iX^i/\sqrt{2}} - e^{-iX^i/\sqrt{2}}\right)\otimes\sigma_1\,, \tag{2.6}$$

in the $(-1)$-picture, where $\Phi$ is the bosonised superconformal ghost, and

$$V_T^{(0)i} = \frac{1}{2}\psi^i\left(e^{i\frac{1}{\sqrt{2}}X^i} + e^{-i\frac{1}{\sqrt{2}}X^i}\right)\otimes\sigma_1\,, \tag{2.7}$$

in the (0)-picture. We first express $V_T^{(0)i}$ in terms of the closed string fields $X^i_{R/L}, \psi^i_{R/L}$ with Neumann boundary condition

$$\begin{aligned} X^i_L &= X^i_R = \frac{1}{2}X^i\,,\\ \psi^i_L &= \psi^i_R = \psi^i\,, \end{aligned} \tag{2.8}$$

at both ends in the NS-sector. In the R-sector (2.8) applies at one end and

$$\psi^i_R = -\psi^i_L\,, \tag{2.9}$$

in the other. The vertex operator (2.7) can be written as

$$\frac{1}{2}\psi^i_R\left(e^{i\sqrt{2}X^i_R} + e^{-i\sqrt{2}X^i_R}\right). \tag{2.10}$$

Next we may fermionise $e^{i\sqrt{2}X^i_R}$ as

$$e^{i\sqrt{2}X^i_R} = \frac{1}{\sqrt{2}}(\xi^i_R + i\eta^i_R) \otimes \Gamma^i \qquad \text{and} \qquad e^{i\sqrt{2}X^i_L} = \frac{1}{\sqrt{2}}(\xi^i_L + i\eta^i_L) \otimes \Gamma^i\,, \tag{2.11}$$

where the cocycles $\Gamma^i$ are introduced to restore the correct commutation relations with the worldsheet fermions [9, 33]. This can be achieved by taking for $\Gamma^i$ the generators of the Spin(4)-Clifford algebra and attaching $\Gamma_5 = \Gamma^{6789}$ to the worldsheet fermions. We complete the transformation by re-bosonising as

$$\frac{1}{\sqrt{2}}\left(\xi^i_{R/L} \pm i\psi^i_{R/L}\right) = e^{\pm i\sqrt{2}\tilde{X}^i_{R/L}} \otimes \tilde{\Gamma}^i\,, \tag{2.12}$$

and attaching a $\tilde{\Gamma}_5$ to $\eta^i_{R/L}$. Here the $\tilde{\Gamma}^i$ form another representation of the Spin(4)-Clifford algebra which commutes with the $\Gamma^i$ representation. With these conventions the fermionic and bosonic currents are then related as

$$\eta^i_R\xi^i_R = i\sqrt{2}\partial X^i_R \qquad \text{and} \qquad \psi^i_R\xi^i_R = i\sqrt{2}\partial\tilde{X}^i_R\,. \tag{2.13}$$

The boundary conditions for the different fields are determined as follows: from (2.7), (2.8) and (2.11) we have

$$\xi^i_L = \xi^i_R = \xi^i \qquad \text{and} \qquad \eta^i_L = \eta^i_R = \eta^i\,. \tag{2.14}$$

In the NS-sector, where $\psi^i_L = \psi^i_R$ on both ends of the open string, (2.14) implies

$$\tilde{X}^i_L = \tilde{X}^i_R = \frac{1}{2}\tilde{X}^i\,, \tag{2.15}$$

i.e. NN boundary conditions for $\tilde{X}^i$. In the R-sector, where $\psi^i_R = \psi^i_L$ at one end and $\psi^i_R = -\psi^i_L$ at the other, (2.14) implies in turn that $\tilde{X}^i_L = \tilde{X}^i_R$ at one end and

$$\tilde{X}^i_L = -\tilde{X}^i_R\,, \tag{2.16}$$

at the other i.e. ND boundary conditions for $\tilde{X}^i$. Using (2.13), (2.15) and (2.16) we may now write

$$\begin{aligned} V_T^{(0)i} &= i\partial_{||}\tilde{X}^i \otimes \sigma_1 \otimes \Gamma_5\Gamma^i\,, \\ V_T^{(0)i} &= i\partial_{\perp}\tilde{X}^i \otimes \sigma_1 \otimes \Gamma_5\Gamma^i\,, \end{aligned} \tag{2.17}$$

in the NS and R-sectors respectively. Finally the vertex operators $V^{(-1)i}$ in the $(-1)$-picture become simply

$$V_T^{(-1)i} = e^{-\Phi}\eta^i \otimes \sigma_1 \otimes \tilde{\Gamma}_5\Gamma^i\,. \tag{2.18}$$

To summarise, in the new variables $(\tilde{X}^i, \eta^i)$ the tachyon vev takes the form of a non-abelian Wilson line in the NS-sector and of a shift in position in the R-sector respectively. In order to obtain the generalisation of (2.17) to non-abelian tachyon vevs all we need to do is to tensor (2.17) with an element of the $u(N)$ Lie algebra generated by $t^a$. For example in the NS-sector we have

$$V_T^{(0)ia} = \partial_{||}\tilde{X}^i \otimes t^a \otimes \sigma_1 \otimes i\Gamma_5\Gamma^i\,. \tag{2.19}$$

## 3. The one loop open string partition function

Here we calculate the one loop partition function for tachyon vevs in open string theory. For simplicity we only consider cases where a single tachyon, $\chi^9$, has a non-vanishing vev. The case where two distinct tachyons have non-vanishing (and therefore anti-commuting) vevs is technically more involved but we do not expect the results to change qualitatively (see also [29]). Although we only need the $x^9$ orbifold radius to be critical, in this section we will assume that the other radii are also critical. We also consider the case of two $\tilde{\mathrm{D}}p$-branes, so that the gauge group is U(2). Therefore there are two cases to consider: either $\chi^9 = v_0 t^0$ or $\chi^9 = v_3 t^3$, where $t^0$ is the identity and $t^1, t^2, t^3$ are the Pauli matrices.

The partition function on the annulus is given by

$$\begin{aligned} Z &= \int_0^\infty \frac{dt}{2t} \mathrm{Tr}_{\mathrm{NS-R}} \left( e^{-2tH_o} \frac{1+(-1)^F}{2} \frac{1+g}{2} \right) \\ &= \int_0^\infty \frac{dt}{2t} \mathrm{Tr}_{\mathrm{NS-R}} \left( e^{-2tH_o} \frac{1+(-1)^F g}{2} \right) \end{aligned} \tag{3.1}$$

Here the second line follows by summing over the $I$ and $\sigma_1$ CP sectors and using the fact that the trace over $((-1)^F + g)$ vanishes [31].

In light-cone gauge and using the conventions of [31], the open string hamiltonian is

$$H_o = \pi \sum_{\mu=0}^{p} p_\mu p^\mu + \pi \sum_{i=6}^{9} p_i p^i + \pi \sum_{\substack{m=0,\\ m\neq \mathrm{l.c.}}}^{9} \left( \sum_{n>0} \alpha_{-n}^m \alpha_n^m + \sum_{r>0} r \psi_{-r}^m \psi_r^m \right) + \pi C_o \,, \tag{3.2}$$

where the third sum is only over the non-light-cone coordinates. By integrating over the momentum in the non-compact directions we arrive at the following expression

$$\begin{aligned} Z &= A_p \int_0^\infty \frac{dt}{t^{(p+3)/2}} \mathcal{A} \\ A_p &= \frac{1}{2^{(p+3)/2}} \int d^{p+1} \left( \frac{k}{2\pi} \right) , \end{aligned} \tag{3.3}$$

where $\mathcal{A}$ is the oscillator sum over all the U(2) CP factors (but not the $I$ and $\sigma_1$ CP factors which were already summed in (3.1)) and the projector $\frac{1}{2}(1 + (-1)^F g)$.

We now consider the annulus correction to the tree-level tachyon potential. The idea is to compute the annulus partition function in terms of the new fields $(\tilde{X}^i, \eta^i)$ where the tachyon background takes the form of a non-abelian Wilson line. We will choose the variables $X^m$, $m \in \{0, \dots, 8\}$ and $\tilde{X}^9$ instead of $X^9$, that is we change variables only in the $X^9$-direction. Similarly we only change the fermionic mode $\psi^9$ to $\eta^9$. We begin by listing the transformations of the various fields under $(-1)^F$

and $g$. We have

$$
\begin{array}{rlll}
(-1)^F: & X^m \to X^m\,; & \psi^m \to -\psi^m\,, & m=0,\dots,8 \\
& \eta^9 \to \eta^9\,; & \tilde{X}^9 \to -\tilde{X}^9\,, & \\
g: & X^m \to X^m\,; & \psi^m \to \psi^m\,, & m=0,\dots,5 \\
& X^j \to -X^j\,; & \psi^j \to -\psi^j\,, & j=6,\dots,8 \\
& \eta^9 \to -\eta^9\,; & \tilde{X}^9 \to -\tilde{X}^9\,. &
\end{array} \tag{3.4}
$$

First we note that the $R$-sector doesn't couple to the tachyon Wilson line. This is already suggested by the tree-level field theory lagrangian (2.3) since no fermions couple to the scalars $\chi^i$. That this is the case in full string theory follows from the observation that the world-sheet fields $\tilde{X}^9$ satisfy ND boundary conditions in the R-sector. As a result, both, the momentum and the winding number vanish in that sector. Consequently a Wilson line representing a tachyon vev has no effect on the Ramond sector. Due to fermionic zero-modes there is also no contribution from the $(-1)^F g$ term in the open string trace and we find (including a factor 4 from the CP labels $t^a$)

$$
4\mathcal{A}_R = -2^{3/2}\frac{f_2^7 f_3}{f_1^7 f_4}\prod_{i=6}^{8}\left(\sum_{n_i} q^{n_i^2}\right) = -8\frac{f_2^4 f_3^4}{f_1^4 f_4^4}\,. \tag{3.5}
$$

Here $f_1(q), f_2(q), f_3(q), f_4(q)$ are the usual oscillator functions (for example see [31]) and $q=e^{-\pi t}$. In (3.5) we used the fact that seven bosons and fermions are unaffected, whereas one boson and one fermion have the opposite modings and that there is no momentum around $x^9$.

On the other hand, if no tachyon vev is turned on the NS sector contribution reads

$$
4\mathcal{A}_{\rm NS} = 2\frac{f_3^8}{f_1^8}\prod_{i=6}^{9}\left(\sum_{n_i} q^{n_i^2}\right) - 2\frac{f_3^3 f_4^5}{f_1^5(\frac{1}{\sqrt{2}}f_2)^3}\left(\sum_{n_9} q^{n_9^2}\right) = 8\frac{f_3^{12}}{f_1^4 f_2^4 f_4^4} - 8\frac{f_3^4 f_4^4}{f_1^4 f_2^4}\,. \tag{3.6}
$$

Here the first term simply comes from eight bosons and fermions with the usual modings and momentum around the orbifold directions. The second terms arises because three bosons and five fermions change sign under $(-1)^F g$. In addition there is only one momentum sum because the zero-modes in the $x^6, x^7, x^8$ directions are projected out. Therefore, at the critical radius and in the absence of tachyon vevs, $4\mathcal{A}_{\rm NS} + 4\mathcal{A}_R = 0$ and hence the one-loop partition function vanishes for all $t$ [31].

To continue we follow [5, 6] and define $(-1)^{\tilde{F}}$ and $(-1)^{\tilde{n}_9}$ by

$$
\begin{array}{rll}
(-1)^{\tilde{F}}: & \psi^a \mapsto -\psi^a & \text{for } a\neq 9 \\
& \eta^9 \mapsto -\eta^9 & \\
& X^a \mapsto X^a\,; \quad \tilde{X}^9 \mapsto \tilde{X}^9 & \\
(-1)^{\tilde{n}_9}: & \psi,\eta \mapsto \psi,\eta & \\
& e^{i\sqrt{2}(n_m X^m + \tilde{n}_9\tilde{X}^9)} \mapsto (-1)^{\tilde{n}_9} e^{i\sqrt{2}(n_m X^m + \tilde{n}_9\tilde{X}^9)} & \text{(no summation on } m\neq 9).
\end{array} \tag{3.7}
$$

| $(-1)^F$ | $(-1)^{n_9}$ | CP$\otimes$cocycle | $\chi_9$-charge |
|---|---|---|---|
| even | even | $t^a \otimes I \otimes I$ | 0 |
| even | odd | $t^a \otimes I \otimes \Gamma^9$ | 2 |
| odd | even | $t^a \otimes \sigma_1 \otimes \Gamma_5$ | 2 |
| odd | odd | $t^a \otimes \sigma_1 \otimes \Gamma_5\Gamma^9$ | 0 |

**Table 1:** Summary of cocycles and the absolute value of the $\chi_9$-charges for an abelian vev $v_0 t^0$.

Consequently we have[3]

$$(-1)^F(-1)^{n_9} = (-1)^{\tilde{F}}(-1)^{\tilde{n}_9}\,. \tag{3.8}$$

Note, however, that individually $(-1)^{\tilde{F}}$ and $(-1)^{\tilde{n}_9}$ act quite differently from $(-1)^F$ and $(-1)^{n_9}$. In particular, $n_9 = 1, F = 0$ is equivalent to $\tilde{F} = 1, \tilde{n}_9 = 0$. Indeed $n_9 = 1$ corresponds to a Wilson line in the dual variable $\tilde{X}^9$, which has $\tilde{F} = 1$ and $\tilde{n}_9 = 0$. Loosely speaking (i.e. at lowest level) the change of variables $(\psi^9, X^9) \to (\eta^9, \tilde{X}^9)$ acts like a $x^9$-momentum $\leftrightarrow$ fermion number duality transformation.

In order to obtain the remaining non-vanishing couplings we first need to identify the various cocycles introduced in the field redefinition. The rules for attaching cocycles to the various operators are straightforward generalisations of those given in [6, 33]. A cocycle $\Gamma_5$ is attached to all $(-1)^F$-odd operators and a cocycle $\Gamma^9$ is attached to $(-1)^{n_9}$-odd states, where $n_9$ is the momentum along $X^9$ in units of $\sqrt{2}$. This cocycle is to restore the usual commutation relations after fermionising and re-bosonising in the $x^9$-direction. As usual, a state will be neutral with respect to $\chi_9$ if its vertex operator commutes with the the Wilson line $v_a t^a \otimes \sigma_1 \otimes \Gamma_5\Gamma^9$ and charged otherwise. As we shall explain in the next two subsections, in order to determine the contribution to the annulus partition function from the charged fields we need to introduce a projection operator $\frac{1}{2}(1-(-1)^{\tilde{F}}(-1)^{\tilde{n}_9})$ into the open string trace. That is the operators

$$P^{\pm} = \frac{1}{4}(1+(-1)^F g)(1 \pm (-1)^{\tilde{F}}(-1)^{\tilde{n}_9})\,, \tag{3.9}$$

project onto the charged or uncharged states depending on whether CP$\otimes$cocyle factors of a state anti-commute or commute with the Wilson line $v_a t^a \otimes \sigma_1 \otimes \Gamma_5\Gamma^9$.

### 3.1 Abelian vacuum expectation values

Let us now consider in detail the case $\chi_9 = \frac{1}{2\sqrt{2}} v_0 t^0$, $\chi_j = 0$, $j \neq 9$. Since $t^0$ commutes with all the $t^a$'s we find the $\chi_9$-charges as in table 1.

[3]The action of $(-1)^{\tilde{F}}(-1)^{\tilde{n}_9}$ on the Fock vacuum $|0\rangle$ is defined as $(-1)^{\tilde{F}}(-1)^{\tilde{n}_9}|0\rangle = (-1)^F(-1)^{n_9}|0\rangle = -|0\rangle$.

Substitution of (3.9) into the NS open string trace leads to the expressions

$$\mathcal{A}_1(v_0,t) = \frac{1}{4}\frac{f_3^8}{f_1^8}\left(\prod_{j=6}^{8}\sum_{n_j\in\mathbb{Z}} q^{n_j^2}\right)\sum_{\tilde n_9\in\mathbb{Z}} q^{(\tilde n_9+v_0)^2} - \frac{1}{4}\frac{f_4^5 f_3^3}{f_1^5(\frac{1}{\sqrt{2}}f_2)^3}\sum_{\tilde n_9\in\mathbb{Z}} q^{(\tilde n_9+v_0)^2} + \tag{3.10}$$

$$+\frac{1}{4}\frac{f_4^8}{f_1^8}\left(\prod_{j=6}^{8}\sum_{n_j\in\mathbb{Z}} q^{n_j^2}\right)\sum_{\tilde n_9\in\mathbb{Z}}(-1)^{\tilde n_9} q^{(\tilde n_9+v_0)^2} - \frac{1}{4}\frac{f_4^3 f_3^5}{f_1^5(\frac{1}{\sqrt{2}}f_2)^3}\sum_{\tilde n_9\in\mathbb{Z}}(-1)^{\tilde n_9} q^{(\tilde n_9+v_0)^2},$$

for the charged states, and

$$\mathcal{A}_0(t) = \frac{1}{4}\frac{f_3^8}{f_1^8}\left(\prod_{j=6}^{8}\sum_{n_j\in\mathbb{Z}} q^{n_j^2}\right)\sum_{\tilde n_9\in\mathbb{Z}} q^{\tilde n_9^2} - \frac{1}{4}\frac{f_4^5 f_3^3}{f_1^5(\frac{1}{\sqrt{2}}f_2)^3}\sum_{\tilde n_9\in\mathbb{Z}} q^{\tilde n_9^2} - \tag{3.11}$$

$$-\frac{1}{4}\frac{f_4^8}{f_1^8}\left(\prod_{j=6}^{8}\sum_{n_j\in\mathbb{Z}} q^{n_j^2}\right)\sum_{\tilde n_9\in\mathbb{Z}}(-1)^{\tilde n_9} q^{\tilde n_9^2} + \frac{1}{4}\frac{f_4^3 f_3^5}{f_1^5(\frac{1}{\sqrt{2}}f_2)^3}\sum_{\tilde n_9\in\mathbb{Z}}(-1)^{\tilde n_9} q^{\tilde n_9^2},$$

for the uncharged states. The various terms in (3.10) and (3.11) correspond to NS, NS$(-1)^F g$, NS$(-1)^{\tilde F}(-1)^{\tilde n_9}$ and NS$(-1)^F g(-1)^{\tilde F}(-1)^{\tilde n_9}$ respectively. In the first term there is no change with respect to the "old variables". In the second term we have used that five fermions and three bosons change sign under $(-1)^F g$ and furthermore that the momentum modes in $x^6, x^7, x^8$ are projected out by $g$ (as in [31], but not for $x^9$). The third and the fourth terms are then obtained by noting that $(-1)^{\tilde F}$ changes the sign of all fermions without any effect on the bosons.

Collecting the different terms we then end up with

$$\mathcal{A} = 4\mathcal{A}_0(t) + 4\mathcal{A}_1(v_0,t) + 4\mathcal{A}_R(t) = \mathcal{A}_A(v_0,t) - \mathcal{A}_A(0,t)\,, \tag{3.12}$$

where $\mathcal{A}_A(v_0,t) = A_1(v_0,t)$ and we have included a factor of 4 from the CP factors $t^a$. This second equality arises from the fact that at $v_0 = 0$ the oscillator sum $\mathcal{A}$ vanishes identically. For later use we also write $\mathcal{A}_A$ in terms of $\theta$-functions

$$\mathcal{A}_A(v_0,t) = e^{-\pi v_0^2 t}\eta^{-12}(it)\theta_3^3(0,it)\Big[\theta_2^4(0,it)\theta_3(itv_0,it) + \theta_4^3(0,it)(\theta_4(0,it) - \theta_3(0,it))\theta_4(itv_0,it)\Big]\,. \tag{3.13}$$

As a check on our result (3.13) let us find the lightest modes which will appear in the field theory approximation. In field theory, the only charged states are the three tachyons $\chi_j$, $j\neq 9$, that is, the states in the $\sigma^1$-sector with $n_9 = 0, (-1)^F$-odd, but $n_j = \pm 1$ for one $j \neq 9$. On the other hand, from the field theory calculation [29] we expect five massless, neutral states; four from the $I$-sector ($(-1)^F$-even) and one from the $\sigma^1$-sector with $n_9 = 1$, $n_j = 0, j\neq 9$. We also expect eight massless fermions which arise from the Ramond sector. Each of these fields also carries 4 degrees of freedom coming from the CP factors of U(2). To obtain these states from the string

| $(-1)^F$ | $(-1)^{n_9}$ | CP⊗cocycle | $\chi_9$-charge |
|---|---|---|---|
| even | even | $t^{0,3} \otimes I \otimes I$ | 0 |
| even | odd | $t^{0,3} \otimes I \otimes \Gamma^9$ | 2 |
| odd | even | $t^{0,3} \otimes \sigma \otimes \Gamma_5$ | 2 |
| odd | odd | $t^{0,3} \otimes \sigma \otimes \Gamma_5\Gamma^9$ | 0 |
| even | even | $t^{1,2} \otimes I \otimes I$ | 2 |
| even | odd | $t^{1,2} \otimes I \otimes \Gamma^9$ | 0 |
| odd | even | $t^{1,2} \otimes \sigma \otimes \Gamma_5$ | 0 |
| odd | odd | $t^{1,2} \otimes \sigma \otimes \Gamma_5\Gamma^9$ | 2 |

**Table 2:** Summary of cocycles and the absolute value of the $\chi_9$-charges for a non-abelian vev $v_3 t^3$.

partition function we need to evaluate the leading order large $t$ behaviour for small $v_0$. From the Ramond sector we find

$$4\mathcal{A}_R = -32 + \mathcal{O}(q)\,, \tag{3.14}$$

whereas the NS sector gives

$$4\mathcal{A}_0 + 4\mathcal{A}_1 = 20 + 12q^{v_0^2} + \mathcal{O}(q)\,, \tag{3.15}$$

i.e. 32 massless fermions, 20 massless bosons and 12 massive bosons. Thus we obtain the correct spectrum of light states.

### 3.2 Non-abelian vacuum expectation values

Let us now consider in detail the case $\chi_9 = \frac{1}{2\sqrt{2}} v_3 t^3$, $\chi_j = 0,\ j \neq 9$. Since $t^3$ does not commute with the other $t^a$'s we find the $\chi^9$ charges given in table 2.

We see that the $t^0$ and $t^3$ sectors contribute exactly the same $\mathcal{A}_0 + \mathcal{A}_1$ terms to the oscillator sum that appeared in the case of an abelian vev, whereas the $t^1$ and $t^2$ sector states contribute similar terms to (3.10) and (3.11) but with opposite projectors

$$\begin{aligned}\mathcal{A}_1'(v_3,t) = &\frac{1}{4}\frac{f_3^8}{f_1^8}\left(\prod_{j=6}^{8}\sum_{n_j\in\mathbb{Z}} q^{n_j^2}\right)\sum_{\tilde n_9\in\mathbb{Z}} q^{(\tilde n_9+v_3)^2} - \frac{1}{4}\frac{f_4^5 f_3^3}{f_1^5(\frac{1}{\sqrt{2}}f_2)^3}\sum_{\tilde n_9\in\mathbb{Z}} q^{(\tilde n_9+v_3)^2} - \\ &-\frac{1}{4}\frac{f_4^8}{f_1^8}\left(\prod_{j=6}^{8}\sum_{n_j\in\mathbb{Z}} q^{n_j^2}\right)\sum_{\tilde n_9\in\mathbb{Z}}(-1)^{\tilde n_9} q^{(\tilde n_9+v_3)^2} + \frac{1}{4}\frac{f_4^3 f_3^5}{f_1^5(\frac{1}{\sqrt{2}}f_2)^3}\sum_{\tilde n_9\in\mathbb{Z}}(-1)^{\tilde n_9} q^{(\tilde n_9+v_3)^2},\end{aligned} \tag{3.16}$$

and

$$\begin{aligned}\mathcal{A}_0'(t) = &\frac{1}{4}\frac{f_3^8}{f_1^8}\left(\prod_{j=6}^{8}\sum_{n_j\in\mathbb{Z}} q^{n_j^2}\right)\sum_{\tilde n_9\in\mathbb{Z}} q^{\tilde n_9^2} - \frac{1}{4}\frac{f_4^5 f_3^3}{f_1^5(\frac{1}{\sqrt{2}}f_2)^3}\sum_{\tilde n_9\in\mathbb{Z}} q^{\tilde n_9^2} + \\ &+\frac{1}{4}\frac{f_4^8}{f_1^8}\left(\prod_{j=6}^{8}\sum_{n_j\in\mathbb{Z}} q^{n_j^2}\right)\sum_{\tilde n_9\in\mathbb{Z}}(-1)^{\tilde n_9} q^{\tilde n_9^2} - \frac{1}{4}\frac{f_4^3 f_3^5}{f_1^5(\frac{1}{\sqrt{2}}f_2)^3}\sum_{\tilde n_9\in\mathbb{Z}}(-1)^{\tilde n_9} q^{\tilde n_9^2}\,.\end{aligned} \tag{3.17}$$

The total contribution to the partition function from the oscillators is thus given by

$$\begin{aligned}\mathcal{A}(v_3,t) &= 2\mathcal{A}_0(t) + 2\mathcal{A}_0'(t) + 2\mathcal{A}_1(v_3,t) + 2\mathcal{A}_1'(v_3,t) + 4\mathcal{A}_R(t)\\ &= \mathcal{A}_{NA}(v_3,t) - \mathcal{A}_{NA}(0,t)\,,\end{aligned} \tag{3.18}$$

where $\mathcal{A}_{NA}(v_3,t) = 2\mathcal{A}_1(v_3,t) + 2\mathcal{A}_1'(v_3,t)$. Again the last equality arises as a consequence of the fact that $\mathcal{A}(0,t) = 0$. In terms of $\theta$-functions $\mathcal{A}_{NA}(v_3,t)$ takes the form

$$\mathcal{A}_{NA}(v_3,t) = e^{-\pi v_3^2 t}\eta^{-12}(it)\theta_3^3(0,it)\theta_2^4(0,it)\theta_3(itv_3,it)\,. \tag{3.19}$$

Again we can check our result by finding the lightest modes which will appear in the field theory approximation. When counting the massless degrees of freedom in the presence of a non-abelian vev we need to take the Higgs mechanism into account. On the other hand we expect thirty-two massless fermions from the Ramond sector. To obtain these states from the string partition function we need to evaluate the leading order large $t$ behaviour for small $v_3$. From the Ramond sector we find

$$4\mathcal{A}_R = -32 + \mathcal{O}(q)\,, \tag{3.20}$$

whereas the NS sector gives

$$\mathcal{A}_0 + \mathcal{A}_1 + \mathcal{A}_0' + \mathcal{A}_1' = 16 + 16q^{v_0^2} + \mathcal{O}(q)\,, \tag{3.21}$$

i.e. 32 massless fermions, 16 massless bosons and 16 massive bosons. This agrees with the counting in the field theory lagrangian (2.3).

## 4. The effective potential

With the preparations of section 3 we are now ready to evaluate the effective tachyon potential,

$$V = -Z\,. \tag{4.1}$$

Before proceeding we note that the integrand $\mathcal{A}$ behaves at small $t$ as

$$\begin{aligned}\mathcal{A} &= 4t^2(\cos(\pi v_0) - 1)(\cos(\pi v_0) - 3) = t^2\mathcal{C}_A\,,\\ \mathcal{A} &= 4t^2(\cos(\pi v_3) - 1)(\cos(\pi v_3) + 1) = t^2\mathcal{C}_{NA}\,,\end{aligned} \tag{4.2}$$

for an abelian and non-abelian vev respectively. Therefore if $p \geq 3$ the integral will diverge. Since there are no open string tachyons (below the critical radius) the integral always converges for large $t$ and $p \geq 0$. Thus we need to regulate the $t$-integral. We do this by introducing a cut-off $\Lambda$ and a corresponding counter term $\mathcal{C}$ (defined in (4.2))

$$\begin{aligned}V_\Lambda &= -A_3\left(\int_\Lambda^\infty \frac{dt}{t^3}\mathcal{A} + \ln\Lambda\mathcal{C}\right),\\ V_\Lambda &= -A_4\left(\int_\Lambda^\infty \frac{dt}{t^{7/2}}\mathcal{A} - 2\Lambda^{-1/2}\mathcal{C}\right),\end{aligned} \tag{4.3}$$

for $p = 3$ and $p = 4$ respectively. This is a particular application of the Fischler-Susskind mechanism [34]. While it is consistent to set the massless closed string fields to zero at tree-level in open string theory this no longer the case at string-loop level where a $\tilde{\mathrm{D}}p$-brane acts as a source for these fields. Ignoring this effect will generically lead to small $t$ divergences in the open string partition function. These divergences can then be compensated by perturbing the sigma model by the vertex operators corresponding to closed string fields. In fact with this in mind, since a $\tilde{\mathrm{D}}p$-brane is a source for closed string fields for any $p$, we should also add these counter terms to the finite cases $p = 0, 1, 2$

$$V_\Lambda = -A_p \left( \int_\Lambda^\infty \frac{dt}{t^{(p+3)/2}} \mathcal{A} + \frac{2}{3-p} \Lambda^{(3-p)/2} \mathcal{C} \right). \tag{4.4}$$

We may then take the limit $\Lambda \to 0$ and obtain a finite effective potential $V_{\text{eff}}$. Of course, for $p = 0, 1, 2$ the addition of the counter term has no effect. We will have more to say about the closed string interpretation and a justification for these choices of counter terms in section 5.

An analytic evaluation of the one loop effective potential does not appear to be possible. To continue we therefore discuss the numerical evaluation of these potentials. In figures 1–10 we have plotted the effective potential, or more precisely $V_{\text{eff}}/A_p$, as a function of the scalar vevs $v_0$ and $v_3$. Since these graphs share similar properties for $p = 0, 1, 2$ and $p = 3, 4$ let us discuss these two cases separately.

First we consider the graphs for $p = 0, 1, 2$. In these cases there is no divergence in the open string integral. In the case of an abelian vev we see that the absolute minimum is at $v_0 = 1$. This corresponds to the $\tilde{\mathrm{D}}p$-branes, which are wrapped over the orbifold, splitting in to D$p$-brane/$\bar{\mathrm{D}}p$-brane pairs sitting at opposite ends of the orbifold,[4] with all branes at one end and anti-branes at the other. We note that $v_0 = 0$ is also a local minimum (this may not be obvious from the plot for $p = 2$ but on closer examination one can see that $V_{\text{eff}}/A_2$ reaches a maxima of about .1 near $v_0 = .11$) so the corresponding states are meta-stable. If we turn on a non-abelian vev then both $v_3 = 0, 1$ are true minima. At $v_3 = 1$, the $\tilde{\mathrm{D}}p$-branes again split into D$p$-brane/$\bar{\mathrm{D}}p$-brane pairs but this time with equal number of branes and anti-branes at each end of the orbifold. In this configuration the oscillator sum vanishes due to Bose-Fermi degeneracy.

Next we consider the cases $p = 3, 4$. Here the form of the effective potential is modified by the appearance of counter terms. We see that for an abelian vev $v_0 = 0$ is now the true minima. Although we note that in the case of a $\tilde{\mathrm{D}}3$-brane there is a very slight local minimum at $v_0 = 1$. Therefore there is a corresponding meta-stable state. If we turn on a non-abelian vev then for the $\tilde{\mathrm{D}}3$-brane $v_3 = 0, 1$ are both minima corresponding to a Bose-Fermi degeneracy. However for the $\tilde{\mathrm{D}}4$-brane the minima

[4] Or, when also counting the compact dimensions, a $\tilde{\mathrm{D}}(p{+}4)$ brane splits into a $\mathrm{D}(p{+}3)/\bar{\mathrm{D}}(p{+}3)$-brane pair.

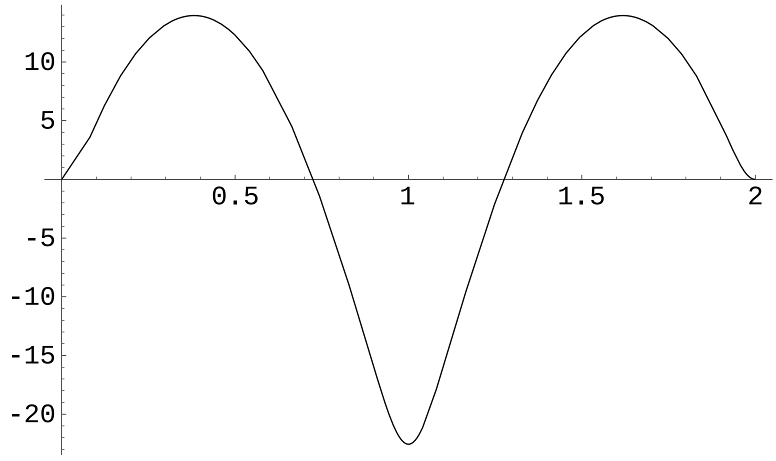


**Figure 1:** The potential on two $\tilde{\mathrm{D}}0$-branes as a function of an abelian tachyon vev.

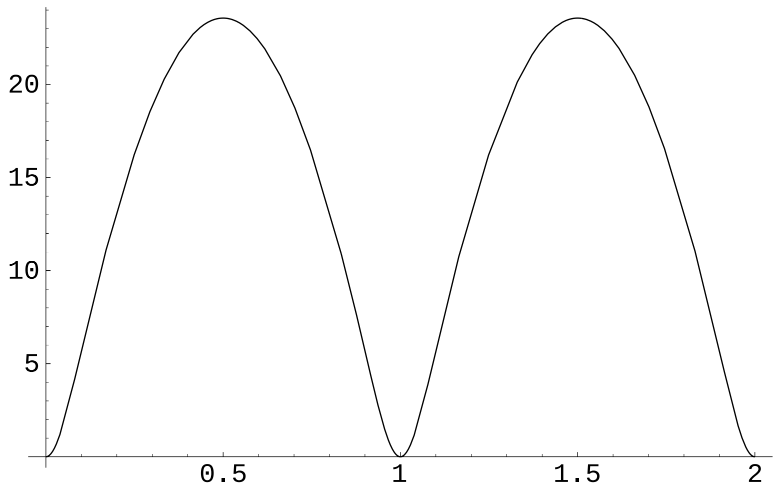


**Figure 2:** The potential on two $\tilde{\mathrm{D}}0$-branes as a function of a non-abelian tachyon vev.

are at $v_3 = 1/2, 3/2$ and there is no Bose-Fermi degeneracy. Note that the change in sign in the effective potential for a non-abelian vev on a $\tilde{\mathrm{D}}4$-brane when compared to the other $\tilde{\mathrm{D}}p$-branes is due to the counter term since the integrand is positive definite. The interpretation of the corresponding states is not clear to us (see also section 5) and furthermore it not clear that the $\tilde{\mathrm{D}}4$-brane is a consistent background for string perturbation theory. Therefore we will not draw any conclusions from the one loop effective potential in this case.

We note that there appears to be a one loop tachyon mass renormalisation. The mass-squared of the tachyon mode $\chi^9$ is now of the form[5]

$$m_9^2 = -\frac{1}{2\alpha'} + \frac{1}{R^2} + \frac{4g_o^2}{\alpha'} V''_{\mathrm{eff}}(0)\,. \tag{4.5}$$

[5]For the sake of clarity we temporarily restore $\alpha'$ and $g_o$.

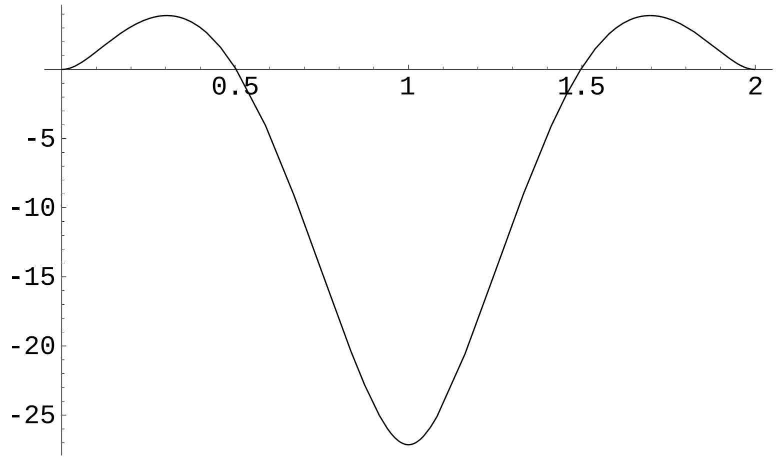


**Figure 3:** The potential on two $\tilde{\mathrm{D}}1$-branes as a function of an abelian tachyon vev.

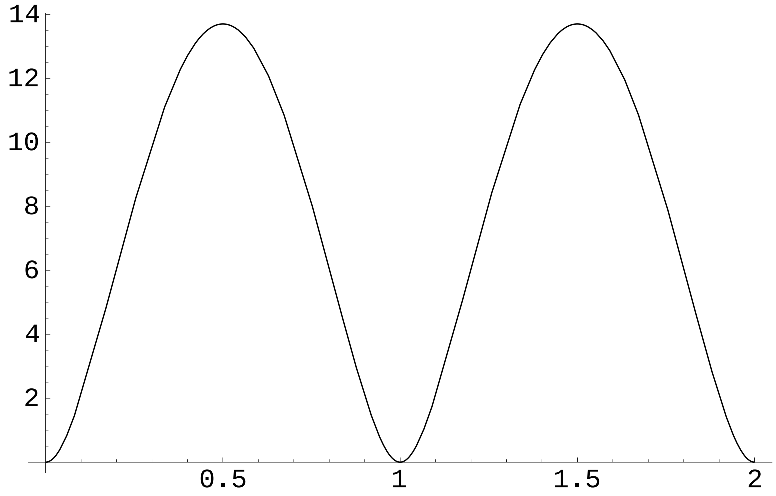


**Figure 4:** The potential on two $\tilde{\mathrm{D}}1$-branes as a function of a non-abelian tachyon vev.

From this we see that the one loop corrected critical radius, i.e. where $m_9^2 = 0$, is

$$R_c = \sqrt{2\alpha'}(1 + 4g_o V''_{\text{eff}}(0))\,. \tag{4.6}$$

Of course this result is only valid provided that $V''_{\text{eff}}(0)$ is finite at $v = 0$. In fact, although $V_{\text{eff}}(v)$ is infinitely differentiable for $v \neq 0, 1$, sufficiently high derivatives diverge at $v = 0, 1$ due to infrared effects in the open string channel. This is the same divergence that occurs in quantum field theory. The second derivative exists for $p = 2, 3, 4$ and one can see that the one loop corrections have increased the critical radius (except for the case of a non-abelian vev on a $\tilde{\mathrm{D}}4$-brane).

In section 3 we assumed for simplicity that all the orbifold radii were critical. However, in order to turn on a single tachyon vev it is only necessary that one direction, which we took to be $x^9$, is critical. In general, if the other radii $R_6, R_7, R_8$

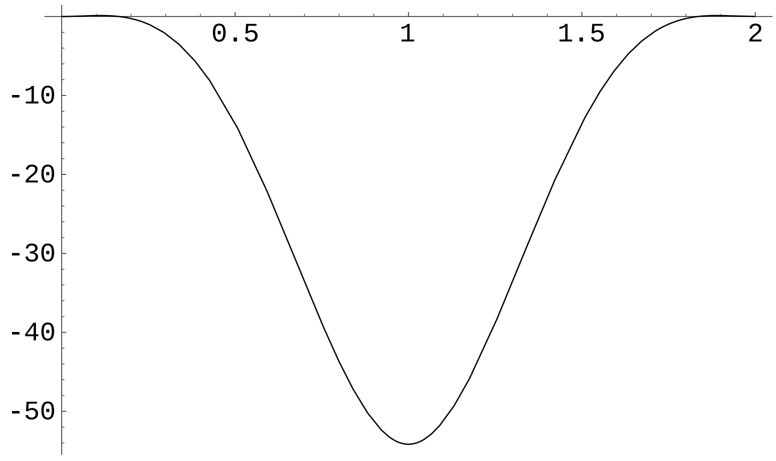


**Figure 5:** The potential on two $\tilde{\mathrm{D}}2$-branes as a function of an abelian tachyon vev.

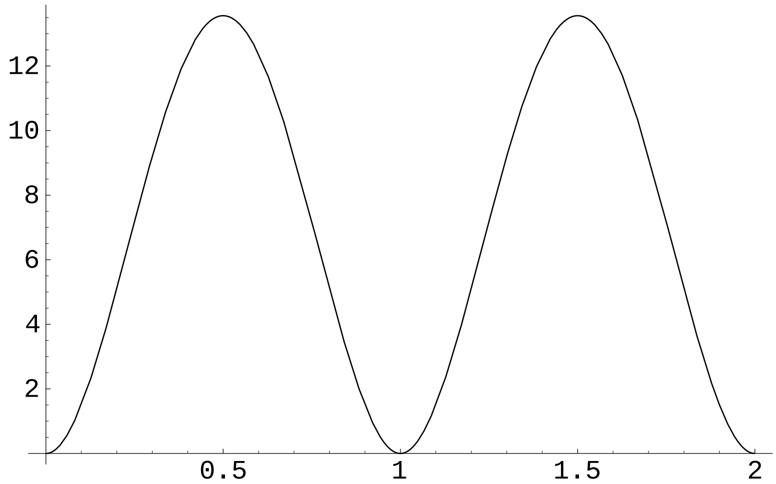


**Figure 6:** The potential on two $\tilde{\mathrm{D}}2$-branes as a function of a non-abelian tachyon vev.



are non-critical the effect in the partition function is to replace the momentum sum

$$\prod_{j=6}^{8}\sum_{n_j\in\mathbb{Z}} q^{n_j^2}\,, \tag{4.7}$$

which appears in (3.10), (3.11), (3.16) and (3.17) by

$$\prod_{j=6}^{8}\sum_{n_j\in\mathbb{Z}} q^{2n_j^2/R_j^2}\,. \tag{4.8}$$

It is easy to check that that $v_0 = 0, 1$ and $v_3 = 0, 1$ are still extrema of the resulting effective potential. We have studied the resulting numerical form for the effective potential for various stable values of these radii and we found that all the minima and maxima which appear in the graphs at the critical radius persist, although the

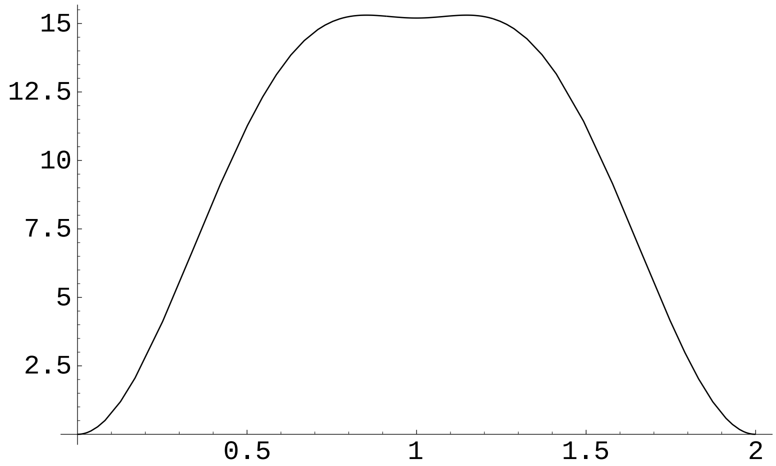


**Figure 7:** The potential on two $\tilde{\text{D}}$3-branes as a function of an abelian tachyon vev.

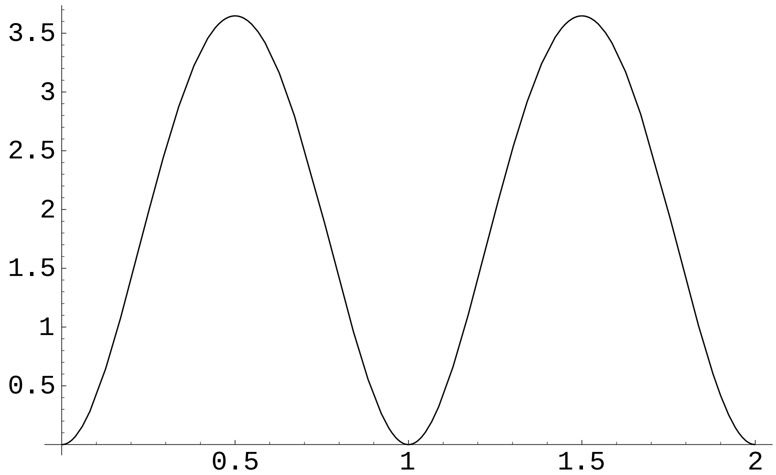


**Figure 8:** The potential on two $\tilde{\text{D}}$3-branes as a function of a non-abelian tachyon vev.



potential no longer vanishes at $v = 0$. We therefore expect that the general shape of the tachyon potential is unaffected for any (stable) values of $R_6, R_7, R_8$. Note that away from the critical radius the authors of [31] found a repulsive force between two non-BPS branes for vanishing tachyon vevs, $\chi^i = 0$. However, we will now show that in the vacuum $v_0 = 1, v_3 = 0$ (which is preferred for $p \le 2$) there is an attractive force and therefore a bound state exists for small $g_o$ and general, but stable, values of $R_6, R_7, R_8$.

Moving the branes in the transverse space spanned by $x^I$ corresponds to turning on a vev $\langle\phi^I\rangle = v^I$. We can see from the disk-level effective action that this is always a marginal deformation in the presence of an abelian tachyon vev $\langle\chi^9\rangle = v_0 t^0$ whereas for a non-abelian vev $\langle\chi^9\rangle = v_3 t^3$ we must have $v^I = v_3^I t^3$. Therefore, in either case, the vev $\phi^I = v_3^I t^3$ that corresponds to separating the branes is a marginal deformation.

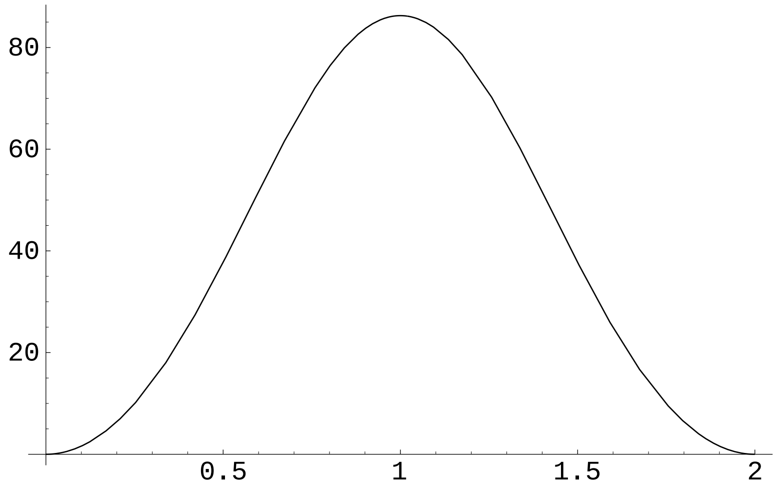


**Figure 9:** The potential on two $\tilde{\mathrm{D}}4$-branes as a function of an abelian tachyon vev.

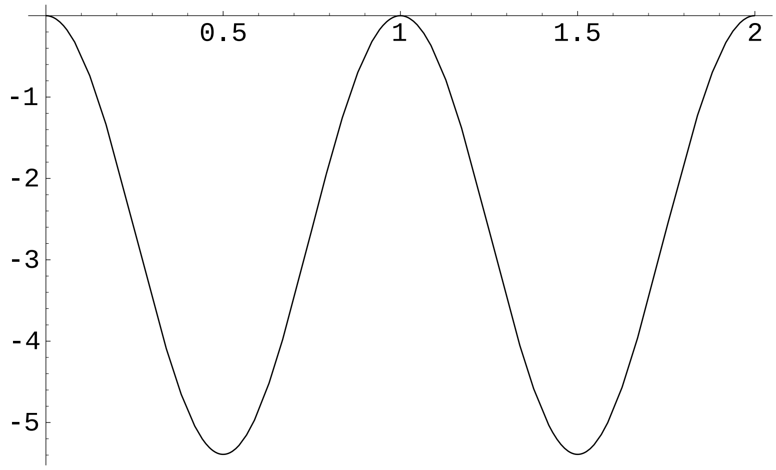


**Figure 10:** The potential on two $\tilde{\mathrm{D}}4$-branes as a function of a non-abelian tachyon vev.

A separation of the branes can be mapped to a Wilson line $\phi_3^I t^3 \otimes I \otimes I$. In the effective potential this corresponds to introducing a multiplicative factor $e^{-r^2 t/2\pi}$, where $r = \sqrt{\sum_I \langle \phi^I \rangle^2}$, for states that don't commute with the Wilson line (i.e. the $t^1, t^2$ CP sectors) whereas the states which do commute (i.e. the $t^0, t^3$ CP sectors) are unaffected. The resulting form for the effective potential is

$$\begin{aligned} V_{\text{eff}} &= -A_p \int_0^\infty \frac{dt}{t^{(p+3)/2}} (2\mathcal{A}_0(t) + 2\mathcal{A}_1(v_0, t) + 2\mathcal{A}_R(t)) e^{-r^2 t/2\pi} + V_0 \,, \\ V_{\text{eff}} &= -A_p \int_0^\infty \frac{dt}{t^{(p+3)/2}} (2\mathcal{A}'_0(t) + 2\mathcal{A}'_1(v_3, t) + 2\mathcal{A}_R(t)) e^{-r^2 t/2\pi} + V'_0 \,, \end{aligned} \tag{4.9}$$

in the cases of an abelian and non-abelian tachyon vev respectively. Here $V_0$ and $V'_0$ are the contributions from the $t^0$ and $t^3$ CP-sectors which do not depend on $r$. Note that now the Ramond sector states do couple to the relevant Wilson line.

For $p = 3, 4$ the integral in (4.9) is divergent since, for small $t$,

$$2\mathcal{A}_0(t) + 2\mathcal{A}_1(v_0, t) + 2\mathcal{A}_R(t) = 2t^2(\epsilon\cos^2(\pi v_0) - 4\cos(\pi v_0) + 7\epsilon - 4) = t^2\mathcal{C}'_A\,, \quad (4.10)$$
$$2\mathcal{A}'_0(t) + 2\mathcal{A}'_1(v_3, t) + 2\mathcal{A}_R(t) = 2t^2(\epsilon\cos^2(\pi v_3) + 4\cos(\pi v_3) + 7\epsilon - 12) = t^2\mathcal{C}'_{NA}\,,$$

where $\epsilon = \sqrt{R_6^2 R_7^2 R_8^2/8}$ and $\epsilon \le 1$ for stable $\tilde{\mathrm{D}}p$-branes. However, we may directly calculate the force between two $\tilde{\mathrm{D}}p$-branes by differentiating (4.9) with respect to $r$. This produces a convergent integral for the force $F_r = -\partial V_{\rm eff}/\partial r$. Furthermore, at least for large $r$, this integral is dominated by the region of small $t$. Therefore we may find a good analytic approximation to the force by simply evaluating

$$\begin{aligned} F_r &= -\frac{r}{\pi}A_p \int_0^\infty \frac{dt}{t^{(p+3)/2}} t^3 \mathcal{C}' e^{-r^2 t/2\pi} \\ &= -\frac{1}{2}A_p (2\pi)^{(3-p)/2}\Gamma(\frac{5-p}{2})\frac{\mathcal{C}'}{r^{4-p}}\,, \end{aligned} \quad (4.11)$$

where $\mathcal{C}'$ is defined in (4.10) for the abelian and non-abelian cases respectively.

Thus the potential corresponding to the force between two non-BPS branes satisfies the laplacian in the transverse space, as one expects for a conservative force. The one loop force $F_r$ is attractive when $\mathcal{C}'$ is positive, repulsive when $\mathcal{C}'$ is negative and vanishes when $\mathcal{C}' = 0$.

We have seen above that the only minima of the effective potential (with the exception of $\tilde{\mathrm{D}}4$-branes) occur at $v_0 = 0, 1$ or $v_3 = 0, 1$. At $v_0 = v_3 = 0$, $\mathcal{C}'_A = \mathcal{C}'_{NA} = 16(\epsilon - 1)$ and the force is repulsive, although it vanishes if all the radii are critical (in agreement with the result of [31]). However at $v_0 = 1$, $\mathcal{C}'_A = 16\epsilon$ and the branes attract each other. Furthermore, for $p = 0, 1, 2$ this is the true vacuum of the system. Thus we find that $\tilde{\mathrm{D}}p$-branes with $p = 0, 1, 2$ are in fact attracted to each other when they are at the minimum of their effective potential. In the case of an non-abelian tachyon vev at $v_3 = 1$, which is a degenerate global minimum along with $v_3 = 0$, $\mathcal{C}'_{NA} = 16(\epsilon - 2)$ and the force is repulsive. Note that at $r = 0$, $\epsilon = 1$ there is a Bose-Fermi degeneracy at $v_3 = 1$, leading to $V_{\rm eff} = 0$, which is removed if the branes are separated.

## 5. Closed string amplitudes

As a independent check of our results for the tachyon potentials found in the last section we now compute the closed string amplitudes at $v_0 = 1$ and $v_3 = 1$ using the boundary state formalism. In particular, in quantum field theory, the effective potential cannot usually be trusted to predict new non-trivial minima. However in string theory, by constructing the corresponding closed string boundary states, we may test the open string predictions. These boundary states will further allow us to identify the massless closed string fields responsible for the small $t$ divergence in the open string partition function in section 3.



JHEP02(2001)018

### 5.1 Boundary state for a D$p$ on $T^4$ and $T^4/g$

According to Sen [2], a $\tilde{\mathrm{D}}p$-brane at $v_0 = 1$ is described by a pair of D$p$-$\bar{\mathrm{D}}p$ branes attached to the orbifold fixed points $x^9 = 0$ and $x^9 = \sqrt{2}\pi$ respectively. In order to construct the corresponding boundary state we first consider a BPS D$p$-brane with $p+1$ infinite dimensions and three dimensions wrapped around $T^4$. This state is given by (A $\bar{\mathrm{D}}p$-brane is obtained by choosing the opposite sign for the RR contribution)

$$|Dp, a, b\rangle = \frac{1}{\sqrt{2}} \left( |p, a, b\rangle_{\mathrm{NSNS}} + |p, a, b\rangle_{\mathrm{RR}} \right) , \tag{5.1}$$

where $a$ and $b$ denote the location of the brane in the infinite transverse dimension and along $x^9$ respectively, i.e.

$$\begin{aligned} |p, a, b\rangle_{\mathrm{NSNS}} &= \mathcal{N} \int dk_{p+1} \cdots dk_5 \sum_{\substack{w_6, w_7, w_8, \\ n_9}} e^{i \frac{n_9}{R_9} b} e^{ika} |p, k, w\rangle_{\mathrm{NSNS}} , \\ |p, a, b\rangle_{\mathrm{RR}} &= 4i\mathcal{N} \int dk_{p+1} \cdots dk_5 \sum_{\substack{w_6, w_7, w_8, \\ n_9}} e^{i \frac{n_9}{R_9} b} e^{ika} |p, k, w\rangle_{\mathrm{RR}} , \end{aligned} \tag{5.2}$$

where $\mathcal{N}$ is a normalisation constant and

$$\begin{aligned} |p, k, w\rangle_{\mathrm{NSNS}} &= \frac{1}{\sqrt{2}} \left( |p, k, w, +\rangle_{\mathrm{NSNS}} - |p, k, w, -\rangle_{\mathrm{NSNS}} \right) , \\ |p, k, w\rangle_{\mathrm{RR}} &= \frac{1}{\sqrt{2}} \left( |p, k, w, +\rangle_{\mathrm{RR}} + |p, k, w, -\rangle_{\mathrm{RR}} \right) , \end{aligned} \tag{5.3}$$

with[6]

$$\begin{aligned} |p, k, w, \eta\rangle_{\mathrm{NSNS/RR}} = \exp\Bigg( & \sum_{n=1}^{\infty} \Bigg[ -\frac{1}{n} \sum_{\substack{\mu=0,\ldots,p, \\ 6,\ldots,8}} \alpha^{\mu}_{-n} \tilde{\alpha}^{\mu}_{-n} + \frac{1}{n} \sum_{\mu=p+1,\ldots,3,9} \alpha^{\mu}_{-n} \tilde{\alpha}^{\mu}_{-n} \Bigg] + \\ & + i\eta \sum_{r>0} \Bigg[ - \sum_{\substack{\mu=0,\ldots,p, \\ 6,\ldots,8}} \psi^{\mu}_{-r} \tilde{\psi}^{\mu}_{-r} + \sum_{\mu=p+1,\ldots,3,9} \psi^{\mu}_{-r} \tilde{\psi}^{\mu}_{-r} \Bigg] \Bigg) |k, \eta\rangle^{(0)}_{\mathrm{NSNS/RR}} . \end{aligned} \tag{5.4}$$

Here $|k, \eta\rangle^{(0)}_{\mathrm{NSNS/RR}}$ denotes the Fock vacuum in the NSNS and RR carrying momentum and winding number $(k_{p+1}, \ldots, k_5, n_9)$ and $(w_6, \ldots, w_8)$ respectively. In the NS sector this uniquely specifies the state. In the RR sector we have

$$\begin{aligned} \psi^{\mu}_{-} |k, -\rangle^{(0)}_{\mathrm{RR}} &= 0 \qquad \text{for } \mu = 0, \ldots, p, 6, \ldots, 8 \\ \psi^{9}_{+} |k, -\rangle^{(0)}_{\mathrm{RR}} &= 0 \qquad \text{for } \mu = p+1, \ldots, 3, 9 \\ |k, +\rangle^{(0)}_{\mathrm{RR}} &= \psi^{9}_{-} \prod_{\substack{\mu=0,\ldots,p, \\ 6,\ldots,8}} \psi^{\mu}_{+} |k, -\rangle^{(0)}_{\mathrm{RR}} . \end{aligned} \tag{5.5}$$

[6] After a double Wick rotation in $x^0$ and $x^4$, so that the directions tangential to the brane are spacelike, we then work in the light cone gauge $x^4 \pm x^5$ (see [35]). For $p = 4$ we undo the Wick rotation and take $x^0 \pm x^1$. The formulas below then have to be changed accordingly.

Next we wish to consider D$p$-branes wrapped over the orbifold $T^4/g$. We begin by displaying the action of the orbifold operation $g$, that is

$$\begin{aligned}
&k^9 \mapsto -k^9, \quad \omega^i \mapsto -\omega^i \qquad \text{for } i = 6,7,8, \\
&\alpha_n^i \mapsto -\alpha_n^i, \quad \tilde{\alpha}_n^i \mapsto -\tilde{\alpha}_n^i, \quad \psi_n^i \mapsto -\psi_n^i, \quad \tilde{\psi}_n^i \mapsto -\tilde{\psi}_n^i, \qquad \text{for } i = 6,7,8.
\end{aligned} \tag{5.6}$$

On the other hand $\mathcal{I}_4$ acts on the RR-sector ground state as

$$\begin{aligned}
&|k_{p+1}, \ldots, k_5, n_9, \omega_6, \omega_7, \omega_8, \eta\rangle \rightarrow \\
&\quad \rightarrow \prod_{i=6}^{9} (\sqrt{2}\psi_0^i) \prod_{i=6}^{9} (\sqrt{2}\tilde{\psi}_0^i) |k_{p+1}, \ldots, k_5, -n_9, -\omega_6, -\omega_7, -\omega_8, \eta\rangle \,.
\end{aligned} \tag{5.7}$$

Furthermore, $(-1)^{F_L}$ changes the sign of the RR ground state so that

$$g|k_{p+1}, \ldots, k_5, n_9, \omega_6, \omega_7, \omega_8, \eta\rangle = |k_{p+1}, \ldots, k_5, -n_9, -\omega_6, -\omega_7, -\omega_8, \eta\rangle \,. \tag{5.8}$$

We therefore conclude that for $b = 0, \sqrt{2}\pi$ the boundary states (5.1) are g-invariant.

Let us now turn to the twisted sector boundary states. The analog of (5.4) in the twisted sector is given by

$$\begin{aligned}
|T, k, \eta\rangle_{\text{NSNS/RR}} = \exp \Bigg( &\sum_{n=1}^{\infty} \Bigg[ -\frac{1}{n} \sum_{\substack{\mu=0,\ldots,p,\\ 6,\ldots,8}} \alpha_{-n}^\mu \tilde{\alpha}_{-n}^\mu + \frac{1}{n} \sum_{\mu=p+1,\ldots,3,9} \alpha_{-n}^\mu \tilde{\alpha}_{-n}^\mu \Bigg] + \\
&+ i\eta \sum_{r>0} \Bigg[ - \sum_{\substack{\mu=0,\ldots,p,\\ 6,\ldots,8}} \psi_{-r}^\mu \tilde{\psi}_{-r}^\mu + \sum_{\mu=p+1,\ldots,3,9} \psi_{-r}^\mu \tilde{\psi}_{-r}^\mu \Bigg] \Bigg) |T, k, \eta\rangle_{\text{NSNS/RR}}^{(0)} \,.
\end{aligned} \tag{5.9}$$

Here the integers $r, n$ take the values

$$\begin{aligned}
n &\in \mathbb{Z}_+ && \text{for } \mu = 0, \ldots, 5 \\
n &\in \mathbb{Z}_+ - \frac{1}{2} && \text{for } \mu = 6, \ldots, 9 \\
r &\in \mathbb{Z}_+ - \frac{1}{2} && \text{for } \mu = 0, \ldots, 5 \\
r &\in \mathbb{Z}_+ && \text{for } \mu = 6, \ldots, 9
\end{aligned} \tag{5.10}$$

in the NSNS-sector, and

$$\begin{aligned}
n &\in \mathbb{Z}_+ && \text{for } \mu = 0, \ldots, 5 \\
n &\in \mathbb{Z}_+ - \frac{1}{2} && \text{for } \mu = 6, \ldots, 9 \\
r &\in \mathbb{Z}_+ && \text{for } \mu = 0, \ldots, 5 \\
r &\in \mathbb{Z}_+ - \frac{1}{2} && \text{for } \mu = 6, \ldots, 9
\end{aligned} \tag{5.11}$$

in the RR-sector. Note that the in the twisted sector the Fock vacuum has no momentum, nor winding number in the orbifold directions. Also, due to the fermionic zero modes in the NSNS sector the the Fock vacuum is defined slightly differently. Concretely, in the NSNS-sector we have

$$\begin{aligned} \psi_-^\mu |T,k,-\rangle^{(0)}_{\rm NSNS} &= 0 \qquad \text{for } \mu = 6,7,8 \\ \psi_+^9 |T,k,-\rangle^{(0)}_{\rm NSNS} &= 0 \qquad \text{for } \mu = p+1,\ldots,3,9 \\ |T,k,+\rangle^{(0)}_{\rm NSNS} &= \prod_{\mu=6,7,8} \psi_+^\mu \psi_-^9 |T,k,-\rangle^{(0)}_{\rm NSNS}\,, \end{aligned} \tag{5.12}$$

whereas in the RR-sector we have

$$\begin{aligned} \psi_-^\mu |T,k,-\rangle^{(0)}_{\rm RR} &= 0 \qquad \text{for } \mu = 0,\ldots,p \\ |T,k,+\rangle^{(0)}_{\rm RR} &= \prod_{\mu=0,\ldots,p} \psi_+^\mu |T,k,-\rangle^{(0)}_{\rm RR}\,. \end{aligned}$$

In $x$-space we then have

$$\begin{aligned} |T,a,\eta\rangle_{\rm NSNS} &= 2\tilde{\mathcal{N}} \int dk_{p+1}\cdots dk_5 e^{ika} |T,k,\eta\rangle_{\rm NSNS}\,, \\ |T,a,\eta\rangle_{\rm RR} &= 4i\tilde{\mathcal{N}} \int dk_{p+1}\cdots dk_5 e^{ika} |T,k,\eta\rangle_{\rm RR}\,, \end{aligned} \tag{5.13}$$

where $\tilde{\mathcal{N}}$ is another normalisation constant. The GSO invariant combination is then given by

$$\begin{aligned} |T,a\rangle_{\rm NSNS} &= \frac{1}{\sqrt{2}} \left( |T,a,+\rangle_{\rm NSNS} + |T,a,-\rangle_{\rm NSNS} \right) \\ |T,a\rangle_{\rm RR} &= \frac{1}{\sqrt{2}} \left( |T,a,+\rangle_{\rm RR} + |T,a,-\rangle_{\rm RR} \right). \end{aligned} \tag{5.14}$$

For a D$p$-brane attached to the orbifold fixed plane $x^9 = 0, \sqrt{2}\pi$ and wrapped around the other orbifold directions we then have 8 twisted sector boundary states $|T_\alpha\rangle$, $\alpha = 1,\ldots,8$.

### 5.2 Boundary states for D$p\bar{\rm D}p$ pairs on $T^4/g$

We are now ready to write down the boundary state for a D$p\bar{\rm D}p$ state with the D$p$ and the $\bar{\rm D}p$ located at the opposite points of the orbifold[7]

$$\begin{aligned} |B_A\rangle \equiv |Dp;\bar{D}p\rangle = {} & \frac{1}{\sqrt{2}} \left( |p,0\rangle_{\rm NSNS} + |p,0\rangle_{\rm RR} \right) + \frac{1}{\sqrt{2}} \left( |p,\sqrt{2}\pi\rangle_{\rm NSNS} - |p,\sqrt{2}\pi\rangle_{\rm RR} \right) + \\ & + \frac{1}{4} \sum_{\alpha=1}^{8} \left( |T_\alpha\rangle_{\rm NSNS} + |T_\alpha\rangle_{\rm RR} \right) - \frac{1}{4} \sum_{\alpha=1}^{8} \left( |\bar{T}_\alpha\rangle_{\rm NSNS} - |\bar{T}_\alpha\rangle_{\rm RR} \right), \end{aligned} \tag{5.15}$$

[7]We temporarily set $a = 0$ and delete this label from the boundary state.

where $T_\alpha, \bar{T}_\alpha$ denote the twisted boundary states located for the D$p$ brane at $b=0$ and the $\bar{\text{D}}p$-brane at $b=\sqrt{2}\pi$ respectively. The relative signs for the various twisted state contributions are fixed by the requirement that the D$p$ and the $\bar{\text{D}}p$-brane have the same twisted sector RR charge [3, 32].

Let us now compare the closed string tree-amplitude for the boundary state (5.15) with the open string loop result (3.12) for $v_0 = 1$. At the critical orbifold radius we have

$$\langle B_A|e^{-lH_c}|B_A\rangle = 4\mathcal{N}^2 l^{\frac{p-5}{2}} \prod_{i=6}^{8}\left(\sum_{w_i\in\mathbb{Z}} e^{-2\pi l w_i^2}\right)\frac{f_2(\tilde{q})^8}{f_1(\tilde{q})^8}\sum_{n_9}(-1)^{n_9} e^{-\pi l \frac{n_9^2}{2}} , \tag{5.16}$$

where $\tilde{q} = e^{-2\pi l}$ and we have used the fact that the twisted sector gives a vanishing contribution to the amplitude. This follows from the fact that

$$_{\text{NSNS}}\langle T_\alpha, \pm|e^{-lH_c}|T_\beta, \pm\rangle_{\text{NSNS}} = -_{\text{RR}}\langle T_\alpha, \pm|e^{-lH_c}|T_\beta, \pm\rangle_{\text{RR}} , \tag{5.17}$$

for $\alpha = \beta$ and vanishes otherwise [3]. A similar relation holds for the twisted sector states $|\bar{T}_\alpha\rangle$ and any amplitude between the two different sets of states always vanishes, i.e. the only non-vanishing amplitudes are between the same two twisted sector states. The closed string partition function for a boundary state $|B\rangle$ is given by

$$Z = \int dl \langle B_A|e^{-lH_c}|B_A\rangle . \tag{5.18}$$

To compare this with the open string loop result (3.12) we use the modular transformation properties (see figures) (with $2l = 1/t$) as well as the duplication formula

$$\theta_2(4\tau) = \frac{1}{2}(\theta_3(\tau) - \theta_4(\tau)), \tag{5.19}$$

leading to

$$\langle B_A|e^{-lH_c}|B_A\rangle = 4\mathcal{N}^2 (2)^{\frac{5-p}{2}} t^{-\frac{p-1}{2}} \eta^{-12}(q)\theta_4^4(0,q)\theta_3^3(0,q)(\theta_3(0,q) - \theta_4(0,q)) . \tag{5.20}$$

This agrees with the open string oscillator sum $\mathcal{A}_A(v_0, t) - \mathcal{A}_A(0, t)$ at $v_0 = 1$ for a suitable choice of the normalisation constant $\mathcal{N}$. Note that to compare the open and closed string partition functions we must also include the factors from the change of the moduli space measure $dl = -dt/2t^2$.

Next we consider a non-abelian vev $v_3 = 1$. This should correspond to a D$p\bar{\text{D}}p$ at $x^9 = 0$ and a $\bar{\text{D}}pDp$ at $x^9 = \sqrt{2}\pi$. The corresponding boundary sate is given by linear superposition. First we consider the situation where the two non-BPS branes sit on top of each other. Then

$$|B_{NA}\rangle = \sqrt{2}|p, 0\rangle_{\text{NSNS}} + \sqrt{2}|p, \sqrt{2}\pi\rangle_{\text{NSNS}} + \frac{1}{2}\sum_{\alpha=1}^{8}|T_\alpha\rangle_{\text{RR}} + \frac{1}{2}\sum_{\alpha=1}^{8}|\bar{T}_\alpha\rangle_{\text{RR}} . \tag{5.21}$$

For a critical orbifold ($R_i = \sqrt{2}$), the corresponding closed string tree-level amplitude is then given by

$$\langle B_{NA}|e^{-lH_c}|B_{NA}\rangle = \frac{4}{l}(8\mathcal{N}^2 - \tilde{\mathcal{N}}^2)\frac{f_3^4(\tilde{q})f_2^4(\tilde{q})}{f_4^4(\tilde{q})f_1^4(\tilde{q})}\,. \tag{5.22}$$

On the other hand we have [31]

$$\tilde{\mathcal{N}}^2 = \frac{16R_9}{R_6R_7R_8}\mathcal{N}^2\,. \tag{5.23}$$

Hence, the amplitude vanishes on a critical orbifold, as expected from the results in section 4.

If the two non-BPS branes are separated a distance $r$ along one of the non-compact directions the corresponding boundary state is

$$|B_{NA}\rangle = |Dp, 0, 0; \bar{\mathrm{D}}p, r, 0; \bar{\mathrm{D}}p, 0, \sqrt{2}\pi; \mathrm{D}p, r, \sqrt{2}\pi\rangle\,. \tag{5.24}$$

The closed string tree amplitude is found to be

$$\begin{aligned}\langle B_{NA}|e^{-lH_c}|B_{NA}\rangle = {} & 4\mathcal{N}^2 l^{\frac{p-5}{2}}\prod_{i=6}^{8}\left(\sum_{w_i\in\mathbb{Z}}e^{-2\pi l w_i^2}\right)\frac{f_2(\tilde{q})^8}{f_1(\tilde{q})^8}\sum_{n_9}(-1)^{n_9}e^{-\pi l\frac{n_9^2}{2}} + \\ & + 4\mathcal{N}^2 l^{\frac{p-5}{2}}e^{-\frac{r^2}{4\pi l}}\prod_{i=6}^{8}\left(\sum_{w_i\in\mathbb{Z}}e^{-2\pi l w_i^2}\right)\frac{f_2(\tilde{q})^8}{f_1(\tilde{q})^8}\sum_{n_9}e^{-\pi l\frac{n_9^2}{2}} - \\ & - 4\tilde{N}^2 l^{\frac{p-5}{2}}e^{-\frac{r^2}{4\pi l}}\frac{f_3^4(\tilde{q})f_2^4(\tilde{q})}{f_4^4(\tilde{q})f_1^4(\tilde{q})}\,.\end{aligned} \tag{5.25}$$

At the critical orbifold radius (5.25) can be rewritten using

$$\theta_3(4\tau) = \frac{1}{2}(\theta_3(\tau) + \theta_4(\tau)), \tag{5.26}$$

leading to

$$\langle B_{NA}|e^{-lH_c}|B_{NA}\rangle = 4\mathcal{N}^2 e^{-\frac{\mathrm{t}r^2}{2\pi}}2^{\frac{5-p}{2}}t^{-\frac{p-1}{2}}\eta^{-12}(q)\theta_4^4(0,q)\theta_3^3(0,q)\left(\theta_4^2(0,q) - \theta_3^2(0,q)\right) + \cdots\,, \tag{5.27}$$

where the ellipsis denotes terms which are independent of the separation $r$. This agrees with the open string result (4.9).

We note that the boundary states $|B_A\rangle$ and $|B_{NA}\rangle$ can exist away from the critical radii. Indeed, considering the small $l$ limit of the closed string amplitudes for arbitrary $R_6, R_7, R_8, R_9$, one see that there are no tachyons in the open string channel provided $R_i \leq \sqrt{2}$ for $i = 6, 7, 8$, as expected from section 4, and $R_9 \geq \sqrt{2}$. However if none of the radii are critical then these boundary states are not connected to the boundary state at $v_a = 0$ by a marginal perturbation (on the disk).

We may again evaluate the force between two boundary states $|B_A\rangle$ and $|B_{NA}\rangle$ for arbitrary but stable values of the radii by simply differentiating the partition function with respect to $r$. At non-zero $r$ the partition function for $|B_A\rangle$ is given in (5.25) whereas for $|B_{NA}\rangle$ the effect on the partition function is simply to include a multiplicative factor of $e^{-r^2/4\pi l}$ in (5.16). Just as in section 4, at large $r$ we may approximate the integral by taking only the leading order large $l$ term in the oscillator, momentum and winding sums. Note that these are independent of the radii and hence the only dependence on the radii enters through $\mathcal{N}$ and $\tilde{\mathcal{N}}$. More explicitly we find

$$\begin{aligned} F_r &= -\frac{64\Gamma(\frac{5-p}{2})}{(4\pi)^{\frac{p-3}{2}}}\frac{\mathcal{N}^2}{r^{4-p}}\,, \\ F_r &= \frac{64\Gamma(\frac{5-p}{2})}{(4\pi)^{\frac{p-3}{2}}}\frac{\mathcal{N}^2(\frac{4R_9}{R_6R_7R_8}-1)}{r^{4-p}}\,, \end{aligned} \tag{5.28}$$

for $|B_A\rangle$ and $|B_{NA}\rangle$ respectively. Now $\mathcal{N}^2 \geq 0$ and hence the force between two states $|B_A\rangle$ is attractive for all stable values of the radii. On the other hand, at the critical radius $R_i = \sqrt{2}$, the force for $|B_{NA}\rangle$ is repulsive. In fact it remains repulsive away from the critical radius as long as $R_9/R_6R_7R_8 \geq 1/4$ which is always the case for stable configurations. The presence of a repulsive force at the critical radius in spite of Bose-Fermi degeneracy is an interesting and maybe unexpected phenomenon.

### 5.3 Interpretation of the counter terms

Using the boundary states found above we can now interpret the counter terms introduced in section 4 by identifying the massless closed string fields exited by these boundary states. For definiteness we consider the case of an abelian vev $v_0$. Consider first the boundary state for a non-BPS D$p$ brane at $v_0 = 0$ [32]

$$|\tilde{D}p\rangle = \frac{1}{\sqrt{2}}|p,0\rangle_{\rm NSNS} + \frac{1}{4}\sum_{\alpha=1}^{8}|T_\alpha\rangle_{\rm RR} + \frac{1}{4}\sum_{\alpha=1}^{8}|\bar{T}_\alpha\rangle_{\rm RR} \tag{5.29}$$

In (5.29) only the untwisted NSNS sector and the twisted RR sector boundary states survive the GSO projection. At long distance, where the massless closed string fields dominate the interaction the twisted RR repulsion works against the untwisted NSNS attraction leading to a no-force condition at the critical radius of the orbifold [31]. At $v_0 = 1$ on the other hand, examination of (5.15) shows that the massless twisted NSNS and RR closed string fields excited by the D$p$ and $\bar{\rm D}p$ boundary states (5.15) live on separate orbifold fixed planes and thus do not interact. Correspondingly they do not contribute to the partition function (5.16). This leaves us with the untwisted contributions from the D$p$ and $\bar{\rm D}p$ boundary states. Naively one might think that this system should be tachyonic. The absence of the tachyonic mode can be understood by noting that the D$p$ and the $\bar{\mathrm{D}}p$-brane are attached to different orbifold fixed planes and can therefore not annihilate. The boundary state (5.15) acts as a dipole source for the untwisted RR-fields and therefore these fields fall off too fast to be relevant at large distance. The NSNS-contributions of the D$p$ and $\bar{\mathrm{D}}p$-brane in (5.15) however, are additive as always. Hence the graviton and dilaton determine the long distance (small $t$) behaviour of $\mathcal{A}_A$ in (3.12). Indeed, the asymptotic behaviour of the dilaton and the components of the graviton for the source (5.15) is expressed in terms of the Green function of the laplacian in the transverse space i.e.

$$\phi, h_{mn} \propto \begin{cases} \dfrac{1}{r^{3-p}}\,; & p \neq 3 \\ \log r\,; & p = 3\,. \end{cases} \tag{5.30}$$

In order to have a consistent string loop expansion we then need to include these massless fields into the tree-level open string amplitude. To first order in $g_c$ this is incorporated by perturbing the flat sigma-model by the vertex operator [34, 41, 40]

$$g_c F(\Lambda) h_{mn} \partial X^m \bar{\partial} X^n\,, \tag{5.31}$$

where $F(\Lambda) = \Lambda^{(3-p)/2}$ for $p \neq 3$ and $F(\Lambda) = \log(\Lambda)$ for $p = 3$. The tree-level (disk) tadpoles induced by the perturbation (5.31) then reproduce the counter terms in (4.2),(4.3) and (4.4). For $p = 0, 1, 2$ these counter terms vanish as $\Lambda \to 0$ and therefore become redundant (see (4.4)). For $p = 3, 4$ on the other hand, the disk tadpoles diverge for $\Lambda \to 0$. However, together with the divergent loop amplitude (4.3) the total amplitude is well defined as $\Lambda \to 0$.

The fact that the metric and dilaton have a large $r$ divergence for $p = 3, 4$ raises the question of whether these states are well defined. For $p = 3$ the logarithmic divergence of the metric is in fact an artifact of perturbation theory. Indeed the situation here is analogous to that for point particles in $2+1$-dimensional gravity. The asymptotic six-dimensional metric of a $\tilde{\mathrm{D}}3$-brane describes a flat spacetime with a wedge cut out

$$ds^2_{(6)} = -\eta_{\mu\nu} dx^\mu dx^\nu + r^{-8Gm}(dr^2 + r^2 d\theta^2)\,. \tag{5.32}$$

Expanding (5.32) in $G$ reproduces the logarithmic correction (5.30). For $p = 4$ the gravitational potential (and the dilaton) increases linearly with the distance. It is therefore not clear to us whether a $\tilde{\mathrm{D}}4$-brane is a meaningful concept beyond tree-level.

### 5.4 Comparison with field theory

In this subsection we compare the running of couplings in the field theory (2.3) with the running of the dilaton for the corresponding boundary state. First we recall that the T-duality of string theory relates some coupling constants that can appear in the effective lagrangian. In addition there is a $\mathbf{Z}_2^4$ symmetry $\chi^i \leftrightarrow -\chi^i$ [29] which is a result of momentum conservation around $x^i$ in the full string theory. In this way, and allowing for field redefinitions, all but the terms involving only $\chi^i$ scalars can be fixed to take the form in (2.3) [29]. For example T-duality and gauge invariance imply that there are no mass terms allowed for the scalars $\phi^I$. However such a non-renormalisation theorem would appear in the field theory in the guise of a hierarchy problem. The remaining terms are less restricted by these symmetries and must have the form

$$\frac{1}{2}m_i^2(\chi^i)^2 + g_1 \sum_{i,j}(\chi^i)^2(\chi^j)^2 + g_2 \sum_{i,j}(\chi^i\chi^j)^2 \,. \tag{5.33}$$

Although string theory determines the couplings $g_1$ and $g_2$ in terms of $g_{\rm YM}$ at tree-level, leading to the potential (2.4), from the field theory perspective there appears to be no symmetry that will ensure the relations between the various couplings is preserved after loop corrections are included.

In what follows we will fix the ambiguities that arise at one loop in quantum field theory by comparing with the string theory results in the previous section. After eliminating the freedom of field redefinitions this leaves the couplings $g_{\rm YM}, g_1, g_2$ and masses $m_i$ which may run independently. Note that quantum field theory is not in general compatible with T-duality so that in principle there may be more independent coupling constants. From the closed string point of view we expect that the couplings $g_{\rm YM}, g_1, g_2$ can be related to some combinations of the dilaton $\varphi$ and components of the metric. We should then be able to understand the running of these couplings through the coupling of the massless closed string fields to the $\tilde{\rm D}p$. It would be interesting to gain a more thorough understanding of this relationship but this is beyond the scope of this paper.

Let us now then consider a $\tilde{\rm D}$3-brane at $v_a = 0$ and T-dualise along the orbifold, so that no dimensions of the brane are wrapped. It is then clear (e.g. see [36]) that the dilaton does not couple and can be taken to be constant. Therefore, one expects that near $v_a = 0$, in the field theory (2.3) $g_{\rm YM}$ does not run at one loop.[8] This can be verified by noting that, since the lagrangian is so similar to that of $N = 4$ Yang-Mills, only graphs with external $\chi^i$'s or fermions are divergent at one loop. At $v_1 = 1$ the $\tilde{\rm D}$3-brane is also described by a field theory approximation. However this field theory is not equivalent to (2.3) with $v_1 = 1$. In particular, the $\tilde{\rm D}$3-brane at $v_1 = 1$ corresponds to wrapped D4/$\bar{\rm D}$4-branes with half a unit of Wilson line on one of them.[9] In this case the dilaton is no longer constant and we expect a running of the couplings. Note that in all cases the open string theory fixes the mass for the scalars $\chi^i$.

[8] Note that if we did not T-dualise, although the corresponding field theory is the same, we would obtain a non-constant dilaton.

[9] This is the T-dual picture of the D6/$\bar{\rm D}$6-branes at opposite ends of the orbifold described above.



JHEP02(2001)018

## 6. Conclusion

In this paper we have determined the open string loop corrections to the flat directions of the tree level potential on critical non-BPS D$p$-branes. The resulting effective potentials in general have multiple minima. Perhaps the most notable result is the observation that for $p = 0, 1, 2$ the global minima occurs at a non-zero vev for the tachyonic modes. Moreover in these vacua parallel $\tilde{\mathrm{D}}p$-branes are attracted to each other, whereas they repel in the unstable vacua. It would be interesting to repeat the analysis of [26] for $\tilde{\mathrm{D}}p$-branes at $v_0 = 1$ to see whether or not a supergravity description is to be expected. We also examined the coupling of closed strings using the boundary state formalism. This allowed us to understand in greater detail the Fischler-Susskind mechanism that is needed to regularise the open string effective potential. Although we discussed some aspects of the relation between the running of the field theory couplings and the closed string fields, a more thorough understanding of this relation would be very desirable in view of a non-supersymmetric generalisation of the AdS-CFT correspondence [27, 28].

Let us summarise our results on the forces between two non-BPS branes and the effective potential of the tachyon modes. We found that the force between two non-BPS branes obtained at $v_a = 0$ is repulsive, except at the critical radius where the force vanishes, as first observed in [31] and explained by Bose-Fermi degeneracy. On the other hand the non-BPS branes obtained at $v_0 = 1$ are attracted to each other, whereas the non-BPS branes obtained at $v_3 = 1$ repel each other (even though there is a Bose-Fermi degeneracy), for all values of the critical radius where the branes are stable. Indeed, so long as only one radius is critical, the absolute minima for $p = 0, 1, 2$ occurs at $v_0 = 1, v_3 = 0$. Therefore, if we consider for example a system of $\tilde{\mathrm{D}}0$-branes at $v_a = 0$ then, although they are stable, they will undergo a phase transition and tunnel into the true minima at $v_0 = 1,\ v_3 = 0$. Curiously, while the $\tilde{\mathrm{D}}0$-branes repel each other at $v_0 = 0$, in the new vacuum at $v_0 = 1$ they are attracted to each other.

In this paper we have concentrated on tachyon kinks along the $x^9$-direction. For this it is enough to set $R_9$ to the critical value. However, if other radii are critical then we can turn on vevs for the corresponding $\chi^i$'s, provided that they all simultaneously anti-commute [29, 30]. In this case we expect local minima at $\chi^I_a = 0, \frac{1}{2\sqrt{2}}$. In addition one may expect new minima with two non-commuting tachyon vevs [29]. In fact the field theory approximation suggest that these vacua are energetically preferred [29]. If so there is an attractive force between the branes since, from (2.5), $\phi^I_a t^a \otimes I \otimes I$ is no longer marginal (except for the centre of mass coordinate $\phi^I_0$). In other words, we expect that the corresponding boundary state is stable (at one loop) and two of these states are attracted to each other. On the other hand the field theory approximation cannot be trusted in this regime and it would therefore be interesting to work out the open string effective potential in this case.

In closing we note that several other combinations of non-BPS branes and stabilising orbifolds can be considered [31, 37, 38, 39, 10]. We expect that the non-abelian flat directions discussed in [29, 30] also exist in these cases. It would be interesting to determine the fate of tachyonic flat directions and the resulting forces in these other examples.

## Acknowledgments

We would like to thank G.M.T. Watts for his initial collaboration on this work and for many insightful comments. We would also like to thank D. Lowe, A. Sen, S. Theisen and A. Tseytlin for helpful discussions. I.S. would like to thank the Erwin Schrödinger Institute in Vienna and also Kings College London for hospitaliy during various stages of this work. N.D.L. has been supported by a PPARC Advanced Fellowship and also acknowledges the PPARC grant PA/G/S/1998/00613.

## References

[1] A. Sen, *Stable non-BPS states in string theory*, *J. High Energy Phys.* **06** (1998) 007 [hep-th/9803194].

[2] A. Sen, *Non-BPS states and branes in string theory*, hep-th/9904207.

[3] A. Sen, *Stable non-BPS bound states of BPS D-branes*, *J. High Energy Phys.* **08** (1998) 010 [hep-th/9805019].

[4] A. Sen, *Tachyon condensation on the brane antibrane system*, *J. High Energy Phys.* **08** (1998) 012 [hep-th/9805170].

[5] A. Sen, SO(32) *spinors of type-I and other solitons on brane-antibrane pair*, *J. High Energy Phys.* **09** (1998) 023 [hep-th/9808141].

[6] A. Sen, *BPS D-branes on non-supersymmetric cycles*, *J. High Energy Phys.* **12** (1998) 021 [hep-th/9812031].

[7] M. Naka, T. Takayanagi and T. Uesugi, *Boundary state description of tachyon condensation*, *J. High Energy Phys.* **06** (2000) 007 [hep-th/0005114].

[8] M. Frau, L. Gallot, A. Lerda and P. Strigazzi, *Stable non-BPS D-branes in type-I string theory*, *Nucl. Phys.* **B 564** (2000) 60 [hep-th/9903123].

[9] J. Majumder and A. Sen, *Vortex pair creation on brane-antibrane pair via marginal deformation*, *J. High Energy Phys.* **06** (2000) 010 [hep-th/0003124].

[10] J. Majumder and A. Sen, *Non-BPS D-branes on a Calabi-Yau orbifold*, *J. High Energy Phys.* **09** (2000) 047 [hep-th/0007158].

[11] A. Sen, *Uniqueness of tachyonic solitons*, hep-th/0009090.

[12] V.A. Kostelecky and S. Samuel, *On a nonperturbative vacuum for the open bosonic string*, *Nucl. Phys.* **B 336** (1990) 263.

[13] A. Sen and B. Zwiebach, *Tachyon condensation in string field theory*, *J. High Energy Phys.* **03** (2000) 002 [hep-th/9912249].

[14] W. Taylor, *D-brane effective field theory from string field theory*, *Nucl. Phys.* **B 585** (2000) 171 [hep-th/0001201].

[15] J.A. Harvey and P. Kraus, *D-branes as unstable lumps in bosonic open string field theory*, *J. High Energy Phys.* **04** (2000) 012 [hep-th/0002117].

[16] N. Moeller and W. Taylor, *Level truncation and the tachyon in open bosonic string field theory*, *Nucl. Phys.* **B 583** (2000) 105 [hep-th/0002237].

[17] N. Berkovits, *The tachyon potential in open Neveu-Schwarz string field theory*, *J. High Energy Phys.* **04** (2000) 022 [hep-th/0001084].

[18] N. Berkovits, A. Sen and B. Zwiebach, *Tachyon condensation in superstring field theory*, *Nucl. Phys.* **B 587** (2000) 147 [hep-th/0002211].

[19] K. Lechner, *A quantum field theory of dyons*, hep-th/0003003.

[20] N. Moeller, A. Sen and B. Zwiebach, *D-branes as tachyon lumps in string field theory*, *J. High Energy Phys.* **08** (2000) 039 [hep-th/0005036].

[21] E. Witten, *Noncommutative tachyons and string field theory*, hep-th/0006071.

[22] A. Sen, *Universality of the tachyon potential*, *J. High Energy Phys.* **12** (1999) 027 [hep-th/9911116].

[23] E. Eyras and S. Panda, *The spacetime life of a non-BPS D-particle*, *Nucl. Phys.* **B 584** (2000) 251 [hep-th/0003033].

[24] Y. Lozano, *Non-BPS D-brane solutions in six dimensional orbifolds*, *Phys. Lett.* **B 487** (2000) 180 [hep-th/0003226].

[25] P. Brax, G. Mandal and Y. Oz, *Supergravity description of non-BPS branes*, *Phys. Rev.* **D 63** (2001) 064008 [hep-th/0005242].

[26] M. Bertolini, P. Di Vecchia, M. Frau, A. Lerda, R. Marotta and R. Russo, *Is a classical description of stable non-BPS D-branes possible?*, *Nucl. Phys.* **B 590** (2000) 471 [hep-th/0007097].

[27] J. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [hep-th/9711200].

[28] I.R. Klebanov and A.A. Tseytlin, *D-branes and dual gauge theories in type-0 strings*, *Nucl. Phys.* **B 546** (1999) 155 [hep-th/9811035].

JHEP02(2001)018

[29] N.D. Lambert and I. Sachs, *Non-abelian field theory of stable non-BPS branes*, *J. High Energy Phys.* **03** (2000) 028 [hep-th/0002061].

[30] N.D. Lambert and I. Sachs, *Non-abelian Born-Infeld action and solitons for critical non-BPS branes*, *J. High Energy Phys.* **08** (2000) 024 [hep-th/0006122].

[31] M.R. Gaberdiel and A. Sen, *Non-supersymmetric D-brane configurations with Bose-Fermi degenerate open string spectrum*, *J. High Energy Phys.* **11** (1999) 008 [hep-th/9908060].

[32] M.R. Gaberdiel, *Lectures on non-BPS Dirichlet branes*, *Class. and Quant. Grav.* **17** (2000) 3483 [hep-th/0005029].

[33] T. Banks, D. Horn and H. Neuberger, *Bosonization of the* SU($N$) *thirring models*, *Nucl. Phys.* **B 108** (1976) 119.

[34] W. Fischler and L. Susskind, *Dilaton tadpoles, string condensates and scale invariance, 2*, *Phys. Lett.* **B 173** (1986) 262.

[35] O. Bergman and M.R. Gaberdiel, *A non-supersymmetric open-string theory and S-duality*, *Nucl. Phys.* **B 499** (1997) 183 [hep-th/9701137].

[36] P.D. Vecchia, M. Frau, I. Pesando, S. Sciuto, A. Lerda and R. Russoet, *Classical p-branes from boundary state*, *Nucl. Phys.* **B 507** (1997) 259 [hep-th/9707068].

[37] M. Mihailescu, K. Oh and R. Tatar, *Non-BPS branes on a Calabi-Yau threefold and Bose-Fermi degeneracy*, *J. High Energy Phys.* **02** (2000) 019 [hep-th/9910249].

[38] M.R. Gaberdiel and J. Bogdan Stefanski, *Dirichlet branes on orbifolds*, *Nucl. Phys.* **B 578** (2000) 58 [hep-th/9910109].

[39] J. Bogdan Stefanski, *Dirichlet branes on a Calabi-Yau three-fold orbifold*, *Nucl. Phys.* **B 589** (2000) 292 [hep-th/0005153].

[40] C.G. Callan, C. Lovelace, C.R. Nappi and S.A. Yost, *String loop corrections to $\beta$-functions*, *Nucl. Phys.* **B 288** (1987) 525.

[41] O.D. Andreev and A.A. Tseytlin, *Partition function representation for the open superstring effective action: cancellation of mobius infinities and derivative corrections to Born-Infeld lagrangian*, *Nucl. Phys.* **B 311** (1988) 205.